Holding the “A” Students Hostage?
College Admissions with Grade Inflation and Noisy Test Scores

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June 2010

Abstract

“Application files are piled high this month in colleges across the country. Admissions officers are poring over essays and recommendation letters, scouring transcripts and standardized test scores. But something is missing from many applications: a class ranking, once a major component in admissions decisions. In the cat-and-mouse maneuvering over admission to prestigious colleges and universities, thousands of high schools have simply stopped providing that information, concluding it could harm the chances of their very good, but not best, students.”


We consider the strategic interplay between the disclosure of class rank information by high schools and the use of noisy standardized exams by colleges. Using a simple, stylized model of college admissions, we show that when high schools are symmetric and competing for the marginal admittance, non-disclosure of class rank may be a (weakly) dominant strategy resulting in a Prisoner’s Dilemma like outcome. But colleges may use the threat of an “SAT only” admissions policy to discipline high schools into providing the relevant class rank information. These threats may be credible under repeated play.

We extend the model to the case where high schools have different student quality reputations and find that more reputable high schools have an even weaker incentive to disclose class ranks, but less reputable high schools a stronger incentive as they are less able to game for the marginal college admittance. This result helps explain why leading high schools, especially private college preparatory schools, are at the vanguard of the class rank non-disclosure movement. We discuss the implications of the current “SAT optional” trend in college admissions on further grade inflation and class rank non-disclosure.

Keywords: College Admissions, Grade Inflation, Class Ranking, SAT

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1 Introduction

Even a cursory inspection of the popular press reveals a growing concern about grade inflation in public high schools and private college preparatory schools. The recent decision by the school board of the prestigious Fairfax County public school system to relax its grading standards garnered national press. *Time Magazine* covered the decision with an article (January 28, 2009) provocatively titled “Virginia Parents Fight for Easier Grading Standards.” Across the Atlantic Ocean, a steep rise in the number of British students earning three *A* grades, the traditional passport to “Oxbridge” admittance, has led Cambridge University to introduce a controversial *A* elite grade requirement for admissions.

A related issue is the decision by some U.S. public high schools and private college “prep” schools not to release class ranks. Many of the schools refusing to disclose class ranks are the schools that traditionally prepare much of the prospective students at selective colleges. For example, in 2009, Highland Park, an elite suburban public school district in Texas, announced its plan to discontinue class ranking beyond the top 10% required by the state (for admissions into the state flagship universities). Highland Park’s peers are considering following suit.1 Similar announcements can be found for schools in other states.2

In each of these scenarios, the concern for colleges is that information provided by the high schools is being diluted, the high school transcript made a less effective screening device for selective college admissions. High school officials retort that colleges should not seek a single quantification of the applicant’s merit and, instead, consider the entirety of the applicant’s high school achievements. But with applications growing much more rapidly than college openings, selective colleges are often unable to devote as much time evaluating an applicant’s high school records as desired by applicants and their high schools. Colleges may de-emphasize the transcript and turn toward other signals of applicant merit, such as standardized test scores:

“Admissions directors say the strategy can backfire. When high schools do not provide enough general information to recreate the class rank calculation, many admissions director say they have little choice but to do something virtually no one wants them to do: give more weight to scores on the SAT and other standardized exams.” *New York Times*, March 5, 2006

In this paper, we consider the strategic interplay between the disclosure of student information by high schools and the use of noisy standardized exams by colleges. Using a simple, stylized model of college admissions, we show that

- When high schools are symmetric and competing for the marginal admittance, non-disclosure of class rank may be a (weakly) dominant strategy resulting in a Prisoner’s Dilemma like outcome

- But colleges may use the threat of an “SAT only” admissions policy or of a “pro class rank” bias to discipline high schools into providing the relevant class rank information

- These threats may be credible under repeated play

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1 See “Other School Districts May Follow Highland Park’s Lead on Class Rank,” *Dallas Morning News*, July 2, 2009

2 e.g. “Schools See Class Rank as a Barrier,” *Columbia Daily Tribune*, February 17, 2010, and “Class Rank is not the Measure of Student, School Decides,” *Minneapolis-St. Paul Star Tribune*, February 3, 2009
Grade inflation and the non-disclosure of class ranks are shown to be principal-agent problems, where the high school holds its “A” students “hostage” in an attempt to extract further admittance for its “B” students. The college may be able to solve this problem through a “Carrot” (rewarding high schools that provide class ranks) or “Stick” (discarding grades entirely in admissions) approach. Both approaches help the college align the incentives of the high schools closer to its own. But both are credible only under repeated play and when the college cares sufficiently about future admissions.

We extend the model to the case where high schools, stochastically, differ in student quality, with the colleges knowing the underlying probabilities but not the actual realizations. This corresponds to the case where one high school has a better student quality reputation than another. We find that the incentive to disclose class ranks are much weaker for the more reputable and stronger for the less reputable high school. Asymmetric reputations across high schools reduce, if not eliminate, the likelihood of a Prisoner’s Dilemma like outcome and increases those of information unraveling, benefitting the college and the more reputable high school at the expense of the less reputable high school.

We use the insights from the basic and extended models to consider the implication of the growing “SAT optional” movement in college admissions. Specifically, we discuss the possible unintended consequences of dropping standardized exams, like the SAT, as an admissions factor on the information quality of the other remaining factors.

2 Basic Framework

We consider a simple, stylized model of college admissions involving two high schools, each with two students applying for admissions to the same college. We abstract away the issue of college competition and of matching between student and college, which are the emphasis of much of the recent economics (theoretical) literature on college admissions. In return, unlike the existing literature, we model the college as having access to more than one screening device; the college receives information from both the high school and a standardized exam administered to all applicants.

The high school perfectly observes the merit of its students but chooses how much information to disclose. The standardized exam noisily measures merit but discloses its information fully. The college decides which three of the four students to accept based on the information it receives. For simplicity, we model students as one of two merit types, “A” or “B,” with “A” students having higher merit. Furthermore, for now, we assume that each high school has one of each type. The college knows this, but it does not know which student at the high school is the “A” student. Under complete information, the college would accept both “A” students and be indifferent toward which “B” student it accepts as

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4High school transcript being potentially more informative than standardized exams (SAT) is consistent with current empirical research. See, for example, Geiser & Santelices (2007) and Niu & Tienda (2009).

5With two student types, grade inflation and class rank non-disclosure are equivalent problems.
the marginal admittee.

The standardized exam yields one of two scores for each student, ‘h’ or ‘l,’ with “h” being the indicator of likely higher merit. “A” students earn a “h” with probability \( \theta_A \) and “B” students with probability \( \theta_B \). The exams are imperfect but informative: \( 0 < \theta_B < \theta_A < 1 \). The high school can credibly disclose the identity of the “A” and “B” students by providing the college a class rank. But it is not required to do so. The high schools make their disclosure decisions simultaneously and without knowledge of the standardized exam outcomes. Figure 1 illustrates this information flow.

![Figure 1: Information Flow](image)

The strategic players in this model are the two high schools and the college. We abstract away strategic play by the students themselves. Payoffs for the three strategic players are as follows:

- **High School**
  - Each “A” student admitted increases utility by \( u_A \), each “B” student by \( u_B \)
  - High School has some “fairness” concerns: \( u_A > u_B \)

- **College**
  - Each “A” student admitted increases utility by \( v_A \), each “B” student by \( v_B \)
  - College cares about the merit of its student body: \( v_A > v_B \)

There are four possible admissions outcomes: (AB,A), (AB,B), (A,AB), (B,AB). Table 1 provides the total payoff earned by each player for each outcome.
The best possible outcome for a high school is both its students getting accepted (payoff of $u = u_A + u_B$) and the worst only its “B” student getting accepted (payoff of $u = u_B$). For the college, the best possible outcome involves both “A” students getting accepted (payoff of $v = 2v_A + v_B$) and the worst only one “A” student getting accepted (payoff of $v = v_A + 2v_B$).

We now consider the one-shot, subgame perfect play for the college and for the high schools, solved using backward induction.

2.1 Strategic Play by College

While the high schools face only a single possible decision node – high schools make their decisions simultaneously, without knowledge of exam scores, and before the college admissions decision – the college faces $4 \times 16 = 64$ possible decision nodes; there are four possible sets of high school disclosure decisions and sixteen possible sets of test score realizations.\(^6\)

But in 48 of the 64 nodes – the nodes involving at least one high school disclosing class ranks – the college can ensure itself its maximum payoff $\bar{v}$ by accepting both students from the high school that does not disclose class ranks and only the “A” student from the high school that does disclose class ranks. If both high schools disclose, the college accepts both “A” students and any one of the “B” students. Standardized exams are immaterial as the class ranking provides sufficient information unraveling. Any other course of action exposes the college to some chance of admitting only one “A” student, possibly lowering the college’s realized utility and definitely its expected utility.

For the 16 nodes where neither high school discloses class ranks, high schools provide no information to the college. The only available screening information is from the standardized exam. Assuming the college is risk neutral, the college maximizes expected payoff by admitting the students with the higher test scores ($h > l$).\(^7\) In case of ties (two or more students scoring “$l$”), the college can use Bayes Rule as a further tie-breaker. With informative standardized exams and three students scoring “$l$,” the college maximizes expected payoff by accepting both students from the high school with two “$l$” scores and only the “$h$” scoring student from the other.\(^8\)

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\(^6\)Disclosure decisions and test scores (conditional on merit) are assumed independent of each other.

\(^7\)Recall that $0 < \theta_B < \theta_A$. Student with “$h$” scores are more likely to be type “A” than “B.” Accepting the “$h$” scores before the “$l$” scores maximizes the probability of accepting both “A” students and, thus, expected utility.

\(^8\)The intuition is similar to that of class ranking: different realized test scores provide an imperfect but informative ranking of the students at the high school. The high school with both students scoring “$l$” is akin to no class ranking. Formally, the probability of the “$h$” scoring student being the “A” given that the other student in the high school scored “$l$” is $\frac{\theta_A(1-\theta_B)}{\theta_A(1-\theta_B)+(1-\theta_A)\theta_B} > \frac{1}{2}$ given $0 < \theta_B < \theta_A$. Eliminating this Bayes Rule tie-breaker does not alter any qualitative result.
is *ex ante* indifferent among the tied students and can randomly ration (equal probability).

The above suggests the following weakly dominant strategy for the college in a one-shot game:

\[
\text{Admissions Decision} = \begin{cases} \\
\text{Accept both from HS2, higher ranked from HS1} & \text{if only HS1 discloses} \\
\text{Accept both from HS1, higher ranked from HS2} & \text{if only HS2 discloses} \\
\text{Accept both higher ranked, randomly select third} & \text{if both disclose} \\
\text{Accept higher test scores, break ties by Bayes Rule} & \text{if neither discloses} \\
\end{cases}
\]

**Figure 2** provides the possible admissions outcome under the above admissions policy, given each high school's class rank disclosure decision.

In a one-shot setting, the college cannot earn a higher payoff using an alternative pure strategy without resorting to non-credible threats. The strategy above maximizes (expected) utility at each possible decision node. Therefore, the payoff from the alternative pure strategy must be no greater or, if greater, must involve a threat that has the college earning less payoff in some decision node (a non-credible threat in the one-shot setting).

There are other weakly dominant pure strategies available to the college, differing in the tie-breakers. We assume that the final ties are broken randomly, with each informationally equivalent applicant given the same chance. But the expected payoff does not differ if a different tie-breaker is used. We maintain random rationing as it is the only “fair” rationing rule (not favoring one high school over another) in a one-shot setting.

### 2.2 Strategic Play by High Schools

**Table 2** provides the expected payoff for each high school given its and its rival’s disclosure decisions and the earlier weakly dominant college admissions strategy. The expectation
is with respect to the unobserved standardized exam outcomes and possible random tie-breakers. The table provides the payoffs to the backward inducted one-shot disclosure game played by the high schools.

\begin{table}[h]
\centering
\begin{tabular}{ccc}
\hline
 & Disclose (D) & High School 2 Not Disclose (ND) \\
\hline
High School 1 Not Disclose (ND) & $u_A + \frac{1}{2}u_B$, $u_A + \frac{1}{2}u_B$ & $u_A, u_B$ \\
\end{tabular}
\caption{High School Expected Payoffs for One-shot Disclosure Game}
\end{table}

The above table uses the following further shorthand notations

\[
\begin{align*}
\theta_{hh} &= \theta_A\theta_B \\
\theta_{hl} &= \theta_A(1 - \theta_B) \\
\theta_{lh} &= (1 - \theta_A)\theta_B \\
\theta_l &= (1 - \theta_A)(1 - \theta_B) \\
\theta^* &= (\theta_{hh} + \frac{1}{2}\theta_{hl} + \frac{1}{2}\theta_{lh})(\theta_{hl} - \theta_{lh})
\end{align*}
\]

The derivation of the payoffs for (Not Disclose, Not Disclose) is provided in the appendix. But the payoff is intuitive. $2\theta^*$ is the difference between the probability of both “A” students being admitted and that of only one, given the college admissions policy and neither high school disclosing class ranks. When the difference is 1 (both “A” students accepted with certainty), $\theta^* = \frac{1}{2}$ and the payoff is $u_A + \frac{1}{2}u_B$, the same as under (Disclose, Disclose). But when the difference is 0, $\theta^* = 0$ and the payoff is $\frac{3}{4}u$, the same as when the college randomly admits students. The payoff depends on the extent to which test scores provide the same sorting information as class ranks. When neither high school discloses, each high school benefits from informative tests (higher $\theta^*$) that shield their “A” students.

When one high school discloses (D), the best response for the other is not to disclose (ND) as $\bar{u} > u_A + \frac{1}{2}u_B$ when $u_B > 0$. Instead of half a chance of getting its “B” student admitted, the high school can guarantee admittance of its “B” student by not disclosing. The college, in order to ensure that it selects both “A” students, is forced to accept both students from the non-disclosing school.

When one high school does not disclose (ND), the best response for the other depends on the value of $u_A$ and $u_B$. Given $0 < \theta_B < \theta_A < 1$, $\theta^*$ is bounded within $(0, \frac{1}{2})$. This implies that the payoff each high school receives when neither school discloses is bounded within $(\frac{3}{4}u, u_A + \frac{1}{2}u_B)$.

- If $u_B > \frac{1}{2}u_A$, $\frac{3}{4}u > u_A$ and not disclose (ND) is the best response to the other high school not disclosing (ND) for any valid $\theta^*$.
- But if $u_B \leq \frac{1}{2}u_A$, there exists some $\theta^* \in (0, \frac{1}{2})$ such that for $\theta^* \in (0, \theta^*)$ disclosing (D) is the best response to the other high school not disclosing (ND) $\theta^*$ is the $\theta^*$ that sets the payoff from neither high school disclosing, $\frac{3}{4}u + \frac{1}{2}(u_A - u_B)\theta^*$, equal with the payoff from disclosing when the other had not, $u_A$, yielding $\theta^* = \frac{u_A - 3aB}{2(u_A - u_B)}$.

The comparison of $u_A$ and $u_B$ is simply a statement of the value high schools attach to “fairness.” The greater $u_A$ is with respect to $u_B$, the more concerned the high school

\[\text{See appendix}\]
is about fairness. When \( u_B > \frac{1}{3} u_A \), the fairness concern is insufficient and there are some valid values of \((\theta_A, \theta_B)\) for which the high school is willing to risk the “A” student in order to increase the chance of both students getting accepted.

\(2\theta^*\) is the probability difference between both “A” students being admitted and only one, given the college admissions policy and neither high school disclosing class ranks. With symmetric high schools, the incidence of this risk of an “A” student being denied admissions is shared evenly. Thus, \(\theta^*\) measures the extent to which the “A” student at a high school is made vulnerable when neither high school discloses. As shown in the Appendix, \(\theta^*\) rises as the standardized exam becomes less noisy: \(\theta_A \to 1\) and \(\theta_B \to 0\). Therefore, more precise standardized exams strengthen the incentive of high schools not to disclose class ranks; more precise exams reduce the vulnerability of the exposed “A” students.

Figure 3 plots the sets of \((u_B, \theta^*)\) for which D and ND, respectively, are the best response to the other high school’s ND. The maximum between \(\{0, \theta^*\}\) provides the boundary between the two sets.

Figure 3: Best Response when Rival H.S. does NOT Disclose

The best response strategy of the high school, in the one-shot setting, can now be summarized as

\[
\text{Best Response} = \begin{cases} 
\text{Not Disclose} & \text{if Rival Discloses} \\
\text{Not Disclose} & \text{if Rival Not Discloses and} \{u_B \geq \frac{1}{3} u_A \text{ or } \theta^* \geq \theta^*\} \\
\text{Disclose} & \text{if Rival Not Discloses and} \{u_B < \frac{1}{3} u_A \text{ and } \theta^* \leq \theta^*\}
\end{cases}
\]

The best response of the high school can be considered a principal-agent problem. As long as \( u_B > 0 \), the incentive faced by the high school (agent) agent deviates from that faced.

[10] When the exam is perfect, \(\theta_A = 1\) and \(\theta_B = 0\), outcome is the same as when both high schools disclose; a perfect exam also provides sufficient information unraveling
by the college (principal). The college wants to accept both “A” students and is indifferent about the identity of the marginal admittee. But the high school, when $u_B > 0$, wants the marginal admittee to be its student. The high school may act against the interest of the college – not disclose its private information – in order to pursue this separate agenda. In a sense, the high school is debating whether to hold its “A” student hostage in an attempt to extract the marginal admittance from the college.

“‘It’s going to depend on the school and the community, but many feel like their students have a better chance of being admitted to a college without class rank,’ said Mel Riddile, an associate director at the National Association of Secondary School principals.” *St. Louis Post-Dispatch*, May 23, 2010 (emphasis added)

The magnitude of the principal-agent problem varies with the magnitudes of two factors: [1] fairness concern [2] actual risk faced by the “A” student from non-disclosure. When the rival high school discloses, there is no risk incurred by the “A” student from its high school not disclosing. As there is no risk, there is also no fairness concern. The principal-agent problem is at its highest in this situation. But when the rival high school does not disclose, there is some risk incurred by the “A” student from non-disclosure. The risk translates into some fairness concern. Depending on the size of the risk ($\theta^*$ value) and the resulting fairness concern ($\theta^*$ and difference between $u_A$ and $u_B$), the principal-agent problem may not be sufficient to induce the high school to act against the college.

### 2.3 Subgame Perfect Equilibrium

Using the best response strategies, we can solve for the subgame perfect equilibrium strategies for the high schools for each possible set of $(u_A, u_B, \theta^*)$ values. Table 3 provides the breakdown

<table>
<thead>
<tr>
<th>$u_B$</th>
<th>$\theta^* &lt; \theta^*$</th>
<th>$\theta^* = \theta^*$</th>
<th>$\theta^* &gt; \theta^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3}u_A$</td>
<td>(D, ND)</td>
<td>(D, ND)</td>
<td>(ND, ND)</td>
</tr>
<tr>
<td></td>
<td>(ND, D)</td>
<td>(ND, D)</td>
<td>(ND, ND)</td>
</tr>
<tr>
<td>$\frac{1}{3}u_A$</td>
<td>(ND, ND)</td>
<td>(ND, ND)</td>
<td>(ND, ND)</td>
</tr>
</tbody>
</table>

When $u_B < \frac{1}{3}u_A$ and $\theta^* < \theta^*$, neither high school is willing to risk its “A” student. The high school will not disclose (ND) only if the other school discloses – thus, no risk. The disclosure game is essentially a variant of the “Chicken” game, with each high school waiting for the other to “blink” and disclose its class ranks. Alternatively, it is a variant of “Hawk-Dove.” As in all “Chicken” and “Hawk-Dove” games, there is a third mixed strategy equilibrium.

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**Table 3: Subgame Perfect (Pure) Strategies for High Schools**

<table>
<thead>
<tr>
<th>$u_B$</th>
<th>$\theta^* &lt; \theta^*$</th>
<th>$\theta^* = \theta^*$</th>
<th>$\theta^* &gt; \theta^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3}u_A$</td>
<td>(D, ND)</td>
<td>(D, ND)</td>
<td>(ND, ND)</td>
</tr>
<tr>
<td></td>
<td>(ND, D)</td>
<td>(ND, D)</td>
<td>(ND, ND)</td>
</tr>
<tr>
<td>$\frac{1}{3}u_A$</td>
<td>(ND, ND)</td>
<td>(ND, ND)</td>
<td>(ND, ND)</td>
</tr>
</tbody>
</table>

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11The subgame perfect equilibrium is not unique, as there are other weakly dominant strategies available to the college. But the qualitative results are similar for these other, similar strategies for the college.

12Alternatively, it is a variant of “Hawk-Dove.” As in all “Chicken” and “Hawk-Dove” games, there is a third mixed strategy equilibrium.
But for all other parameter values, the high school is willing to risk its “A” student for additional admittance. Excepting the knife’s edge case of $\theta^* = \theta^*$, not disclosing (ND) is a dominant strategy for all other parameter values. The outcome of the high schools playing their dominant non-disclosure strategy is the classic Prisoner’s Dilemma result: the high schools end up with an expected payoff less than what they could have earned if they had, instead, both played their dominated strategy of class rank disclosure (D).

When neither high school discloses, the expected payoff each high school earns is $\frac{3}{4}u + \frac{1}{2}(u_A - u_B)\theta^*$. But as discussed earlier, $\theta^* \in (0, \frac{1}{2})$. Thus, the expected payoff when neither high school discloses is bounded within $(\frac{3}{4}u, u_A + \frac{1}{2}u_B)$. The expected payoff each high school earns when they both disclose is the upper bound $u_A + \frac{1}{2}u_B$. The high schools are made worse off by jointly playing their dominant strategy vis-à-vis jointly playing their dominated strategy.

Intuitively, this is because joint non-disclosure always leads to some chance, however small, of some “A” student being denied admission. Therefore, it is not surprising to see that the college is also made worse off when the high schools jointly non-disclose compared to jointly disclose: $\bar{v} > \frac{1}{2}(\bar{v} + v) + \frac{1}{2}(\bar{v} - v)\theta$ where $\bar{v} = 2v_A + v_B$ and $v = v_A + 2v_B$. When both high schools disclose, there is information unraveling. But when neither does, the high schools provide no screening for the college, making the college worse off.

“We’re seeing 30, 40 valedictorians at a high school because they don’t want to create these distinctions between students,” said Jess Lord, dean of admission and financial aid at Haverford College in Pennsylvania. ‘If we don’t have enough information, there’s a chance we’ll become more heavily reliant on test scores, and that’s a real negative to me.’” Washington Post, November 18, 2006

The Prisoner’s Dilemma result provides an alternative interpretation of the reluctance of high schools to disclose class rank: reluctance is driven less by greed and more by fear that, by disclosing, the high school may be disadvantaging its “B” students against those of its non-disclosing rivals. The high school would prefer to disclose its class rank if its rival would as well but it cannot trust its rival to do so. In this interpretation, non-disclosure may be the “fair” strategy, as non-disclosure is not so much holding the “A” student hostage as it is refusing to hurt the “B” student in order to help the “A” student.

As the college is also made worse off by the Prisoner’s Dilemma result, there may be some coordination among the college and high schools that may enable all players to earn a higher expected payoff. We consider such coordination in the next section. For simplicity, we limit our discussion there to the relevant case of $u_B \geq \frac{1}{3}u_A$.

3 Repeated Strategic Play

The root of the Prisoner’s Dilemma result is the implicit punishment the college levies against high schools that disclose class rank or, alternatively, the implicit reward the college provides high schools that refuse to disclose. In its desire to ensure that it admits both “A” students, the college creates disincentives for the high school to provide relevant

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13 Derivation of the expected payoff when neither high school discloses is similar to the derivation of the associated expected payoffs for the high schools.
screening information. The college and high schools can escape the Prisoner’s Dilemma if they can eliminate these disincentives.

Commitment to one of the following two alternative college admissions policies would eliminate the disincentives

\[
\text{“Carrot” Admissions} = \begin{cases} 
\text{Disclose (D)} \\
\text{Not Disclose (ND)}
\end{cases}
\]

Under either alternative, the college threatens the high school with an “SAT only” admissions policy if it does not provide class ranks. In the “Carrot” version, the high school that does disclose is made no worse off; its “A” student is always accepted. But in the “Stick” version, all high schools are punished when any one refuses to disclose.

**Tables 4 and 5** provide the expected payoffs for the high schools when the college is able to commit to the “Carrot” and “Stick” alternatives, respectively.

<table>
<thead>
<tr>
<th>High School 2</th>
<th>Disclose (D)</th>
<th>Not Disclose (ND)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>School 1</strong></td>
<td>( u_A + \frac{1}{2} u_B, u_A + \frac{1}{2} u_B )</td>
<td>( \bar{u}, \frac{1}{2} \bar{u} + \frac{1}{2} (\theta_A - \theta_B)(u_A - u_B) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High School 2</th>
<th>Disclose (D)</th>
<th>Not Disclose (ND)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>School 1</strong></td>
<td>( \frac{3}{4} \bar{u} + \frac{1}{4} (u_A - u_B)\theta^<em>, \frac{3}{4} \bar{u} + \frac{1}{4} (u_A - u_B)\theta^</em> )</td>
<td>( \frac{3}{4} \bar{u} + \frac{1}{4} (u_A - u_B)\theta^<em>, \frac{3}{4} \bar{u} + \frac{1}{4} (u_A - u_B)\theta^</em> )</td>
</tr>
</tbody>
</table>

We have shown earlier that \( u_A + \frac{1}{2} u_B > \frac{3}{4} \bar{u} + \frac{1}{2} (u_A - u_B)\theta^* \). This implies that, under “Stick” Admissions, disclosure (D) is a weakly dominant strategy for both high schools. Both (D,D) and (ND,ND) are Nash equilibria, but only (D,D) is stable. This occurs as non-disclosure by any high school eliminates, for all high schools, the ability to influence marginal admittance and exposes all “A” students to risk. There is no longer any gain to not disclosing, only risk.

Under “Carrot” Admissions, disclosure is a strictly dominant strategy as \( u_A + \frac{1}{2} u_B > \frac{1}{2} \bar{u} + \frac{1}{2} (\theta_A - \theta_B)(u_A - u_B) \) and \( \bar{u} > \frac{1}{2} \bar{u} + \frac{1}{2} (u_A - u_B)\theta^* \). The first inequality stems from \( 0 < (\theta_A - \theta_B) < 1 \), the second from \( \theta^* \in (0, \frac{1}{2}) \). The only pure strategy Nash equilibrium involves both high school disclosing class ranks (D,D), resulting in full information unraveling. The incentives faced by the high schools are now more aligned with the college. The high school can only try to game marginal admittance by disclosing its private information.
With either alternative admissions policy, full information unraveling is the stable Nash equilibrium outcome. The expected payoff earned by the college and high schools are higher under “Carrot” or “Stick” Nash equilibria than under the earlier subgame perfect equilibrium. But the Nash equilibrium from these “SAT only” admissions policies rest on non-credible threats.

On the equilibrium path, the college is allowed to evaluate students using grades (class ranks) first and then test scores. This maximizes the college’s current payoff as class ranks are more informative than noisy test scores. But off the equilibrium path, the college must discard all class rank information and evaluate strictly on the noisy test scores. In a one-shot setting, this commitment is not credible as ignoring class ranks necessarily lowers the expected payoff for the college. High schools who realize this will refuse to acknowledge the threat and proceed not to disclose class ranks.

``SAT Only” Admissions

\[
\begin{array}{c|c|c}
\text{Test Scores} & \text{Test Scores} & \text{Test Scores} \\
\text{Grades then} & \text{Just Exams} & \text{Just Exams} \\
\text{Exams} & \text{Exams} & \text{Exams} \\
\end{array}
\]

Figure 4: Equilibrium and Off Equilibrium Paths

However, the Folk Theorem suggests that the threat may be credible in a repeated setting. Under repeated play, the college may resist the temptation of current gain in order to ensure future class rank information. Specifically, the college may play the “Carrot” or “Stick” Admissions strategy repeatedly and the high schools the following trigger strategy:

\[
\text{High School Repeated Play Strategy} = \begin{cases} 
\text{Disclose} & \text{if College has always stuck to stated admissions policy} \\
\text{Not Disclose} & \text{otherwise} 
\end{cases}
\]

The high school threatens to revert to infinite Prisoner’s Dilemma if the College fails to punish any non-disclosing high school, ever. For sufficiently high discount factor and infinite play, this counter threat by the high schools ensures the credibility of the college’s “SAT
only” threat to the high schools.\textsuperscript{14}

To see this, recall that the college would only be tempted to deviate from “Carrot” or “Stick” when one and only one high school discloses its class ranks.\textsuperscript{15} The college earns an expected payoff of $\frac{1}{2}(\bar{v} - \bar{v}) + \frac{1}{2}(\bar{v} - \bar{v})$, the expected payoff from admissions using only test scores, when it stays with “Carrot” or “Stick.” But the college could earn the higher expected payoff of $\bar{v}$ if it cheats and uses the forbidden class rank information. The college will resist temptation if the contemporary gain from cheating is less than the value of class rank information in all later admissions.

\[
\bar{v} - \frac{1}{2}(\bar{v} - \bar{v}) + \frac{1}{2}(\bar{v} - \bar{v})\theta^* \leq \sum_{t=1}^{T} \delta^t (\bar{v} - \frac{1}{2}(\bar{v} - \bar{v}) + \frac{1}{2}(\bar{v} - \bar{v})\theta^*)
\]

Gain from Cheating ≤ Loss from Cheating

For infinite play ($T \to \infty$), the above inequality satisfies for discount factor $\delta \geq \frac{1}{2}$. On the equilibrium path, the high schools will always disclose. But the counter threat by the high schools ensures that the college, for discount factor $\delta \geq \frac{1}{2}$, will carry out its “SAT only” threat in an off equilibrium node. Additionally, each high school, on an off equilibrium node where the college had earlier failed to punish non-disclosure, will play $\text{ND}$ given its Nash conjecture on the other high school. Therefore, for sufficiently high discount factor, the “Carrot” and “Stick” alternative admissions policies are sustainable (subgame perfect equilibrium) with infinitely repeated play.\textsuperscript{16}

The high schools are willing to coordinate with the college on this repeated subgame perfect equilibrium as the expected payoff from $(\text{D, D})$ is higher than that for $(\text{ND, ND})$. By holding the college to its threat of “SAT only” in the event of non-disclosure, the high schools are able to eliminate each other’s opportunistic behavior. The college holds the current “A” students “hostage” and the high schools the future “A” students. As long as the college cares sufficiently about future admissions, a standstill that benefits all three strategic players results. The “A” students are also made better off, but at the explicit cost of the “B” students.

4 Extension: School Heterogeneity

In the basic model, we assumed symmetric high schools. We now extend the model to allow for some high school heterogeneity. Students are stochastically allocated to the two high schools, with one high school having a greater probability of drawing more “A” students than the other. This extension allows us to consider the impact of school reputation on the strategic interplay between college and high schools.

\textsuperscript{14}This scheme assumes that high schools are able to monitor the college adequately – e.g. they observe realized test scores, disclosure decisions, and admissions outcome at the end of the admissions period

\textsuperscript{15}When both high schools disclose, the college is able to use all of its information under either “Carrot” or “Stick.” When neither high discloses, there is no class rank information that tempts

\textsuperscript{16}However, this set-up is not renegotiation-proof. An analogous renegotiation-proof set-up may be derived following, e.g., Abreu (1988) or van Damme (1989)
4.1 High School Reputation

Each high school has at least one “A” student. The second student may be “A” or “B,” stochastically determined. The probability of the second student also being “A” is $\gamma_1$ for high school 1 and $\gamma_2$ for high school 2. High school 1 has a higher reputation student body: $\gamma_1 > \gamma_2$. The above information is common knowledge. But the actual realized merit of each student is private knowledge, known only by the high school. For simplicity, we also assume that each high school also knows the realized merit composition of its rival.

As before, each student takes an informative but noisy standardized exam ($0 < \theta_B < \theta_A < 1$), the results of which are made available to the college, and each high school decides whether to disclose class ranks. But a high school may disclose class ranks only if the second student is merit “B” as, otherwise, there is no meaningful distinction between the students. Therefore, the decision not to disclose may either be a choice or a consequence of the high school having two “A” students. The high school cannot credibly signal nor the college effectively screen between the two scenarios.

“When you have 10 students and they’re all working really, really hard, they may all be very, very close in GPA, but it’s a statistical number,’ Rollins said, explaining that a tiny fraction of a percentage point can make the difference among students with high grades.” Dallas Morning News, July 2, 2009

This model extension allows us to consider the claim by officials at leading high schools and college preparatory schools that class ranks impose an artificial distinction among similarly high merit students and that their refusal to disclose class ranks is, in effect, an expression of this ranking challenge.

4.2 Strategic Play by College

Class ranks, when disclosed, still remain the most informative signal. When only one high school discloses class ranks, the expected payoff maximizing decision of the college, in a one-shot setting, is still to admit both students from the non-disclosing and the higher ranked student from the disclosing high school; this ensures the college its maximum utility as the college admits all the students who could be “A” – the lower ranked student who is denied admission is identified as “B” by the class ranking. When both high schools disclose, the college selects both higher ranked students and is indifferent about the third admittee, a choice between “B” students at the two high schools. As before, disclosure by any high school still provides sufficient information unraveling.

But when neither high school discloses, the college is no longer left with just test scores. The college may also consider the high school attended by the students. With $\gamma_1 > \gamma_2$, the two high schools are asymmetric. Ceteris paribus, a student from high school 1 is more likely to be an “A” than a student from high school 2. There is no longer any need for random rationing; in case of ties, admission of the the student from high school 1 maximizes expected payoff for the college.

\^17With four students vying for three slots, the assumption of each high school having at least one “A” student is largely innocuous, as some student is always admitted from each high school.
Moreover, attended high school may be more informative than test score. Using Bayes rule and assuming independence between attended high school and test score:

\[
\Pr(\text{being "A" | "T" test score, HS1 attendance}) = \frac{\gamma_1 (1 - \theta_A)}{\gamma_1 (1 - \theta_A) + (1 - \gamma_1) (1 - \theta_B)}
\]

\[
\Pr(\text{being "A" | "h" test score, HS2 attendance}) = \frac{\gamma_2 \theta_A}{\gamma_2 \theta_A + (1 - \gamma_2) \theta_B}
\]

The first probability is greater than the second when

\[
\frac{\gamma_1}{1 - \gamma_1} \frac{\gamma_2}{1 - \gamma_2} > \frac{\theta_A}{1 - \theta_A} \frac{\theta_B}{1 - \theta_B}
\]

\[\text{Info Content of HS Attendance \quad Info Content of Test Score}\]

\((\ast)\)

The information content of attended high school increases as \((\gamma_1 - \gamma_2) \to 1\), maximal asymmetric reputation. Similarly, the information content of test score increases as \((\theta_A - \theta_B) \to 1\).

When neither high school discloses class rank and the above inequality \((\ast)\) holds, the college maximizes its expected payoff by admitting both students from the more “reputable” high school and the higher test score student from the less “reputable” high school; attending high school 1 trumps scoring “h” on the standardized exam. This suggests the following weakly dominant strategy for the college in this extended model (one-shot):

\[
\text{Admissions Decision} = \begin{cases} 
\text{Accept both from HS2, higher ranked from HS1} & \text{if only HS1 discloses} \\
\text{Accept both from HS1, higher ranked from HS2} & \text{if only HS2 discloses} \\
\text{Accept both higher ranked, randomly select third} & \text{if both disclose} \\
\text{Accept both from HS1, higher test score from HS2} & \text{if neither discloses and (\ast)} \\
\text{Accept higher test scores, HS1 for ties} & \text{if neither discloses and not (\ast)}
\end{cases}
\]

where \((\ast)\) indicates satisfaction of the above inequality.

When some high school discloses class ranks, there is no difference in the one-shot admissions strategy between the basic and extended model. But when neither high school discloses, there is a difference; the more reputable high school is favored over the less reputable high school, the extent to which depends on the difference in information content between attended high school and test score.\(^{18}\)

### 4.3 Strategic Play by High School

The favoritism shown by the college to the more reputable high school affects the backward inducted (one-shot) best response strategies of both high schools. Tables showing the underlying expected payoffs, the expectation with respect to test scores, for possible student combinations – \((AA,AA), (AA,AB), (AB,AA), (AB,AB)\) – are provided in the appendix. The expected payoffs for all disclosure decision pairs except \((ND,ND)\) are the same as in the basic model, as any disclosure still provides sufficient information unraveling, but the expected payoffs for \((ND,ND)\) differs.\(^{19}\)

\(^{18}\)A sufficient condition for the inequality to hold is for \(\gamma_1 > \theta_A\) and \((\gamma_1 - \gamma_2) > (\theta_A - \theta_B)\)

\(^{19}\)The derivation of the expected payoff for (Not Disclose, Not Disclose) for the extended model is in the appendix. \(2u_A\) substitutes for \(u\) for student combinations involving a school with two “A” student
When the information content of attended high school is greater than that of test score, (⋆) satisfied, test scores play no admissions role and the backward-inducted (one-shot) best response strategies of the high schools are unambiguous. The more reputable high school (high school 1) can ensure admittance for both its students by not disclosing (ND) but opens itself to some chance of admissions denial by disclosing (D). Not disclosing (ND) is a strictly dominant strategy for the more reputable high school under (⋆).

For the less reputable high school (high school 2), not disclosing (ND) is the best response to rival’s disclosing under (⋆), as not disclosing (ND) ensures admissions for both students while disclosing (D) introduces some chance of admissions denial. And disclosing (D) ensures that admittee will be the “A” student while not disclosing (ND) does not.

When the information content of attended high school is less than that of test score, (⋆) not satisfied, test scores do still play an admissions role. The backward-inducted best response strategies of the high school, as in the basic model, depends on the “fairness” concern and the risk faced by the “A” student by non-disclosure – functions of $u_A$, $u_B$, $\theta_A$, and $\theta_B$. Table 6 provides a comparison of the expected payoff, when neither high school discloses class ranks, between the basic and extended not (⋆) model, for the comparable case of (AB,AB).

<table>
<thead>
<tr>
<th>Test Score</th>
<th>High School 1</th>
<th>High School 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic</td>
<td>Extended Not (⋆)</td>
</tr>
<tr>
<td>h h , h h</td>
<td>$\frac{3}{4} \bar{u}$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>h h , h l</td>
<td>$\bar{u}$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>h h , l h</td>
<td>$\bar{u}$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>h h , l l</td>
<td>$\bar{u}$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>h l , h h</td>
<td>$u_A$</td>
<td>$u_A$</td>
</tr>
<tr>
<td>h l , h l</td>
<td>$u_A \frac{1}{2} u_B$</td>
<td>$u_A \frac{1}{2} u_B$</td>
</tr>
<tr>
<td>h l , l h</td>
<td>$u_A \frac{1}{2} u_B$</td>
<td>$u_B$</td>
</tr>
<tr>
<td>h l , l l</td>
<td>$u_A$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>l h , h h</td>
<td>$u_B$</td>
<td>$u_B$</td>
</tr>
<tr>
<td>l h , h l</td>
<td>$\frac{1}{2} u_A + u_B$</td>
<td>$u_B + \frac{1}{2} u_A$</td>
</tr>
<tr>
<td>l h , l h</td>
<td>$\frac{1}{2} u_A + u_B$</td>
<td>$u_B + \frac{1}{2} u_A$</td>
</tr>
<tr>
<td>l h , l l</td>
<td>$u_B$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>l l , h h</td>
<td>$\frac{1}{2} \bar{u}$</td>
<td>$\frac{1}{2} \bar{u}$</td>
</tr>
<tr>
<td>l l , h l</td>
<td>$\bar{u}$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>l l , l h</td>
<td>$\bar{u}$</td>
<td>$\bar{u}$</td>
</tr>
<tr>
<td>l l , l l</td>
<td>$\frac{1}{2} \bar{u}$</td>
<td>$\bar{u}$</td>
</tr>
</tbody>
</table>

Note: $\bar{u} = u_A + u_B$

For the six test score realizations involving only one high school having two “h” scoring students, the expected payoffs are the same between the basic and extended not (⋆) models; the test scores strictly determine admissions. But for the remaining ten realizations, the expected payoffs are weakly higher (eight higher, two same) for the more reputable high school and weakly lower (eight higher, two same) for the less reputable high school in the

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20Table 6 consolidates the information in Tables A1 and A7 in the appendix
extended not (⋆) model, as attended high school matters in admissions.

This indicates that the ex ante expected payoff from (Not Disclose, Not Disclose) – the probability weighted sum of the expected payoff for each test score realization – is higher for high school 1 and lower for high school 2 in the extended not (⋆) model – with (AB, AB) as the realized student combination. The incentive faced by high school 1 not to disclose (ND) is stronger and that faced by high school 2 weaker in the extended model. Reputation helps insulate high school 1 from some of the risk faced by non-disclosing but reduces the ability of high school 2 to game for the marginal admittance.

### 4.4 Subgame Perfect Equilibrium and Repeated Play

When the information content of attended high school is greater than that of test score, (⋆) satisfied, the one-shot subgame perfect equilibrium involves the college playing some variant of the admissions policy described earlier, the more reputable high school 1 not disclosing class ranks, and the less reputable high school 2 disclosing class ranks. Both the college and high school 1 achieve their maximum payoff, at the explicit cost of high school 2. Strong asymmetric reputation eliminates the ability of high school 2 to game for marginal admittance, leading to it disclosing class ranks in order to ensure its “A” student admission.

But when the information content of attended high school is less than that of test score, (⋆) not satisfied, a Prisoner’s Dilemma like outcome is still possible for some set of valid \((u_A, u_B, \theta_A, \theta_B)\) values. But the incentives driving toward a Prisoner’s Dilemma outcome are weaker in the extended model; the more reputable high school faces greater expected payoff under (ND, ND) and the less reputable high school less in the extended model.\(^{21}\) There are more values of \((u_A, u_B, \theta_A, \theta_B)\) that lead to the information unraveling outcome of (ND, D) when high schools have even weakly asymmetric reputation.

The college benefits from asymmetric reputations in the one-shot setting but suffers under repeated play. With symmetric reputation, the college can use an “SAT only” admissions threat to elicit class rank disclosure from both high schools if the college is sufficiently concerned about future admissions. This is shown in our discussion of the basic model under repeated play – with symmetric reputation, attended high school has no admissions value and neither high school has an advantage over the other. All three players have a desire to coordinate an escape out of Prisoner’s Dilemma similar to that in the basic model.

With asymmetric reputation, the reputable high school is less inclined to coordinate with the college and not at all if the information content of attended high school is greater than that of test score. Additionally, punishment schemes involving infinite reversion are infeasible as punishment may be triggered when the high school cannot disclose (as opposed to chooses not), as when both students are “A” types.\(^{22}\) Coordination based on a punishment scheme involving temporary reversion may (but not necessarily) be possible – something akin to Green & Porter (1984) – but as the punishment is weaker than for an infinite reversion, the discount factor required to satisfy incentive compatibility will be higher.

\(^{21}\)Expected utility from disclosing when the rival does not is the same under both models. So the difference between disclosing and not disclosing when the rival does not disclose is less under the extended model.

\(^{22}\)Recall that we assume that neither the high school can signal nor the college screen between the two non-disclosure scenarios.
If coordination is not possible, the college may be able to elicit class rank information from the less reputable high school, as in the one-shot subgame perfect equilibrium, but not from the more reputable high school. This implies that when the less reputable high school has a better realized student body than the more reputable high school, \((AB,AA)\), college admissions will be suboptimal. A “B” student will be the marginal admittee over an available “A” student. When reputation is sufficiently asymmetric \((\gamma_1 \gg \gamma_2)\), this can occur even if the “B” student has a lower test score than the excluded “A” student.

“Nearly a decade after some of the state’s top-performing high schools began dropping class rank from their students’ transcripts, more are following their lead ... They follow a number of high schools in the state and nationwide that have dropped class rank, including Whitefish Bay, Shorewood and Greendale high schools, as well as a host of private college prep schools in the Milwaukee area.” *Milwaukee-Wisconsin Journal Sentinel*, May 28, 2010

These results provide some insight into why leading high schools, especially elite private college preparatory schools, have been at the vanguard of the class rank non-disclosure movement. Once a high school has established a sufficient lead in student quality reputation, it has a strong incentive to stop disclosing class rank and force the college to use high school reputation as an admissions factor. Moreover, among similarly reputable high schools, the decision of one not to disclose can cascade into the others also choosing not to disclose, as non-disclosure is often the best response to non-disclosure among similar schools. A more formal analysis of this dynamic intuition, perhaps through an extension of the model with endogenously accumulated school reputation, would seem fruitful. We leave such an analysis for future research.

5 Conclusion

The developed models are not meant to be general. Some of the results are knife-edge, driven by judicious choice of assumptions. Similar results can be derived from more general but complicated models. Our particular stylized models were chosen so as to illustrate, simply but with some rigor, a key point that is often lost in the college admissions discussions: elimination of noisy signals of student merit, such as the SAT and other standardized exams, can have adverse impact on the information content of other signals.

When high schools have incentives that deviate from those of the college – such as when concern about the marginal admittance outweigh those of “fairness” to the “A” student – grade inflation and non-disclosure of class ranks can result as principal-agent problems. The high school may not disclose (hold its “A” student hostage) in order to extract further admissions from the college. Alternatively, the high school may not disclose in order to ensure that its “B” student is not hurt by the opportunistic behavior of rival high schools. Under either interpretation, the outcome is the same: both high schools reduce their screening information, making the high schools and college worse off in a Prisoner’s Dilemma sense.

The threat of resorting to an “SAT only” admissions policy, credible under repeated play, may help discipline high schools into disclosing their private information on the students. But the converse is also policy relevant. Without noisy standardized exams, the college may not be able to coordinate an escape from the Prisoner’s Dilemma. Threatening
to admit students randomly or blindly favor some high school would also, in theory, work. But both threats would most likely be viewed as being maliciously capricious, making the threats politically infeasible.

Furthermore, in a world with asymmetric reputation among high schools, the elimination of the use of standardized exams as an admissions factor could force colleges to weigh more heavily high school reputation in admissions. This would, in turn, provide a stronger incentive for more reputable high schools not to disclose their class ranks and/or inflate their grades. Students at more reputable high schools would benefit at the explicit cost of those in less reputable high schools. The already intense competition to get into the “right high schools” would intensify even more.

The conclusion we draw is not that standardized exams, such as the SAT, are the “gold standard” among admissions factors; in fact, in our models, we allow class ranks and possibly even high school reputation to be more informative signals. Rather, we show how treatment of one admissions factor can have consequences, intended and unintended, on the information quality of other admissions factors. An important caveat to our concluding discussion, one raised by proponents of the “SAT optional” admissions movement, is that the noise in standardized exams may not be innocuous. The models assume that the exam noise differed across student types, but not high schools. A key criticism of the SAT and related standardized exams is that the exams favor student characteristics other than academic merit, such as socio-economic status.

In terms of our model, this would be, for example, the extension where the probability of a student scoring “h” was lower for those in one high school than the other, across both merit types. Such an extension may lead to results qualitatively similar to those of our asymmetric reputation extension. The high school favored by the exam faces a stronger incentive not to disclose while the disadvantaged high school a weaker incentive. With large enough exam bias, the subgame perfect equilibrium could be driven to (Not Disclose, Disclose), allowing the college and the advantaged high school to benefit at the explicit cost of the disadvantaged high school. Elimination of the exam would reduce the bias against the disadvantaged high school but also introduce informational problems that could lead to “A” students not being admitted – suggesting a classic “equity versus efficiency” trade-off.

Lastly, the developed models abstract away issues of college competition and of matching by assuming a single college. The choice allows us to keep the studied strategy space tractable. However, in doing so, we ignore some interesting and possibly relevant dynamics. Perhaps most salient is that of colleges setting their admissions policy strategically not only to extract information from high schools but also to increase their share of “A” students. For example, a college may coordinate with a high school to admit both the “A” and “B” students from that high school. The college ensures the admission of the “A” student at the cost of admitting the “B.” The high school becomes a “feeder” school for the college. We consider this future research.
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Appendix

Derivation of (Not Disclose, Not Disclose) Payoff for Basic Model

When neither high school discloses, the only source of screening information is the standardized exam. Given \(0 < \theta_B < \theta_A < 1\), the college maximizes its expected payoff by admitting the students with the higher test scores first. Table A1 provides, for each of the sixteen possible exam realizations, the probability of the realization, the possible admissions decision, and the expected payoff to High School 1 for that realization, with expectation over possible ties.

<table>
<thead>
<tr>
<th>(A,B),(A,B)</th>
<th>Probability</th>
<th>Possible Admissions</th>
<th>Expected Payoff (HS1, College)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h,h),(h,h)</td>
<td>(\theta_A\theta_B \times \theta_A\theta_B)</td>
<td>ALL</td>
<td>(\frac{1}{2}u, \frac{1}{2}(\bar{v} + v))</td>
</tr>
<tr>
<td>(h,h),(h,l)</td>
<td>(\theta_A\theta_B \times \theta_A(1 - \theta_B))</td>
<td>(AB,A)</td>
<td>(u_A, \bar{v})</td>
</tr>
<tr>
<td>(h,h),(l,l)</td>
<td>(\theta_A\theta_B \times (1 - \theta_A)\theta_B)</td>
<td>(AB,B)</td>
<td>(u_A, v)</td>
</tr>
<tr>
<td>(h,l),(h,h)</td>
<td>(\theta_A(1 - \theta_B) \times \theta_A\theta_B)</td>
<td>(AB,AB)</td>
<td>(u_A + \frac{1}{2}u_B, \bar{v})</td>
</tr>
<tr>
<td>(h,l),(h,l)</td>
<td>(\theta_A(1 - \theta_B) \times (1 - \theta_A)\theta_B)</td>
<td>(AB,AB)</td>
<td>(u_A + \frac{1}{2}u_B, \frac{1}{2}(\bar{v} + v))</td>
</tr>
<tr>
<td>(l,l),(h,h)</td>
<td>((1 - \theta_A)\theta_B \times \theta_A\theta_B)</td>
<td>(B,AB)</td>
<td>(u_B, \bar{v})</td>
</tr>
<tr>
<td>(l,l),(h,l)</td>
<td>((1 - \theta_A)\theta_B \times \theta_A(1 - \theta_B))</td>
<td>(AB,AB)</td>
<td>(u_B + \frac{1}{2}u_A, \frac{1}{2}(\bar{v} + v))</td>
</tr>
<tr>
<td>(l,l),(l,l)</td>
<td>((1 - \theta_A)\theta_B \times (1 - \theta_A)\theta_B)</td>
<td>(B,AB)</td>
<td>(u_B, \frac{1}{2}(v + \bar{v}))</td>
</tr>
</tbody>
</table>

For a given realization of test scores, each of the possible admissions are equally likely given the assumed tie-breaker. The \textit{ex ante} expected payoff, across all realizations, is the sum of the expected payoffs for each realization above weighted by the probability of the realization. It is the same for High School 1 and 2, by symmetry. This weighted sum can be expressed as \(\frac{1}{4}u + \frac{1}{2}(u_A - u_B)\theta^*\), with the notation defined as in the body of the text.

When \(\theta_A = \theta_B\), \(\theta^* = 0\) and the \textit{ex ante} expected payoff simplifies to \(\frac{1}{4}u\), which is the expected payoff for a high school when the college randomly (but with equal probability) admits students: \(\frac{1}{2}(u + \bar{u} + u_A + u_B) = \frac{3}{4}u\). With \(\theta_A = \theta_B\), the standardized exam has no screening value and admissions by test score is equivalent to random admissions.

The \textit{ex ante} expected payoff for the college, across all test score realizations, when neither high school discloses can be derived in a similar manner, yielding \(\frac{1}{2}(\bar{v} + v) + \frac{1}{2}(\bar{v} - v)\theta^*\).
Derivation of $\theta^* \in (0, \frac{1}{2})$

$\theta^*$ can be expanded in terms of $\theta_A$ and $\theta_B$ as follows

$$
\theta^* = (\theta_{hh} + \frac{1}{2}\theta_{hl} + \frac{1}{2}\theta_{lh} + \theta_{ll})(\theta_{hl} - \theta_{lh})
= \left[ (\theta_A\theta_B + \frac{1}{2}(1 - \theta_A)\theta_B + (1 - \theta_A)(1 - \theta_B)) \right] (\theta_A(1 - \theta_B) - (1 - \theta_A)\theta_B)
= \left[ \frac{1}{2}(\theta_A + \theta_B) + \theta_A\theta_B \right] (\theta_A - \theta_B)
$$

The first order conditions for $\theta^*$ with respect to $(\theta_A, \theta_B)$ are

$$
\frac{\partial \theta^*}{\partial \theta_A} = 1 - \theta_A + 2\theta_A\theta_B - \theta_B^2 > 0 \quad \text{and} \quad \frac{\partial \theta^*}{\partial \theta_B} = -1 + \theta_B - 2\theta_A\theta_B + \theta_A^2 < 0
$$

and the cross-derivative: $\frac{\partial^2 \theta^*}{\partial \theta_A \partial \theta_B} = 2(\theta_A - \theta_B) > 0$

The derivatives show that $\theta^*$ achieves its maximum when $\theta_A \to 1$ and $\theta_B \to 0$ and its minimum when the difference between $(\theta_A - \theta_B) \to 0$, recalling that $0 < \theta_B < \theta_A < 1$. As $\theta_A \to 1$ and $\theta_B \to 0$, $\theta^* \to \frac{1}{2}$. As $(\theta_A - \theta_B) \to 0$, $\theta^* \to 0$.

Intuitively, $2\theta^*$ is the difference in probability between the admissions outcome involving both “A” students being admitted and only one “A” student being admitted. With test scores being informative, $0 < \theta_B < \theta_A < 1$, this probability difference is positive and bounded within $(0, 1)$. Thus, $\theta^*$ is bounded within $(0, \frac{1}{2})$. The probability difference is maximized when test scores are perfect ($\theta_A = 1, \theta_B = 0$) and minimized when test scores are just noise ($\theta_A = \theta_B$). Thus, $\theta^* \to \frac{1}{2}$ as $\theta_A \to 1$ and $\theta_B \to 0$ and $\theta^* \to 0$ as $(\theta_A - \theta_B) \to 0$.

**Expected Payoff Tables for Extended Model**

| Table A2: High School Expected Payoffs, One-shot Extended Model, (AA,AA) |
|---------------------------|---------------------------|
| High School 1 | High School 2 |
| High  | D | ND |
| School 1 | n/a | n/a |
| D | 2$u_A$, $u_A$ | 2$u_A - (1 - \theta_A^2)\theta_A^2 u_A$, $u_A + (1 - \theta_A^2)\theta_A^2 u_A$ if (*) |

| Table A3: High School Expected Payoffs, One-shot Extended Model, (AA,AB) |
|---------------------------|---------------------------|
| High School 1 | High School 2 |
| High  | D | ND |
| School 1 | 2$u_A$, $u_A$ | n/a |
| D | 2$u_A - (1 - \theta_A^2)\theta_A^2 u_A$, $\frac{1}{2}(\theta_A + \theta_B)(u_A - u_B)$ if (*) |

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Table A4: High School Expected Payoffs, One-shot Extended Model, (AB,AA)

<table>
<thead>
<tr>
<th>School</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(AB)</td>
<td></td>
<td>θ</td>
<td></td>
</tr>
<tr>
<td>(AA)</td>
<td></td>
<td>θ</td>
<td></td>
</tr>
</tbody>
</table>

Table A5: High School Expected Payoffs, One-shot Extended Model, (AB,AB)

<table>
<thead>
<tr>
<th>School</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(AB)</td>
<td></td>
<td>θ</td>
<td></td>
</tr>
<tr>
<td>(AA)</td>
<td></td>
<td>θ</td>
<td></td>
</tr>
</tbody>
</table>

Derivation of (Not Disclose, Not Disclose) Payoff for Extended Model

Table A6: (Not Disclose, Not Disclose) Possible Admissions (Extended Model)

<table>
<thead>
<tr>
<th>Test Score</th>
<th>Probability</th>
<th>(AB,AB)</th>
<th>(AA,AB)</th>
<th>(AB,AA)</th>
<th>(AA,AA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>h , h</td>
<td>θ₁,h₁ × θ₂,h₁</td>
<td>(AB,AB)</td>
<td>(AA,AB)</td>
<td>(AB,AA)</td>
<td>(AA,AA)</td>
</tr>
<tr>
<td>h , h</td>
<td>θ₁,h₁ × θ₂,h₁</td>
<td>(AB,AB)</td>
<td>(AA,AB)</td>
<td>(AB,AA)</td>
<td>(AA,AA)</td>
</tr>
<tr>
<td>h , h</td>
<td>θ₁,h₁ × θ₂,h₁</td>
<td>(AB,AB)</td>
<td>(AA,AB)</td>
<td>(AB,AA)</td>
<td>(AA,AA)</td>
</tr>
<tr>
<td>h , h</td>
<td>θ₁,h₁ × θ₂,h₁</td>
<td>(AB,AB)</td>
<td>(AA,AB)</td>
<td>(AB,AA)</td>
<td>(AA,AA)</td>
</tr>
</tbody>
</table>

Admissions for the case where (⋆) does not hold

For (AA): θ₁,₁h₁ = θ₁,h₁ = θₐ(1 - θₐ) θ₁,₁h₁ = (1 - θₐ)²

For (AB): θ₁,₁h₁ = θₐθₐ, θ₁,h₁ = θₐ(1 - θₐ) θ₁,₁h₁ = (1 - θₐ)θₐ, θ₁,₁h₁ = (1 - θₐ)(1 - θₐ)
Table A7: (Not Disclose, Not Disclose) Expected Payoff (Extended Model)

<table>
<thead>
<tr>
<th>Test Score</th>
<th>Probability</th>
<th>(AB,AB)</th>
<th>(AA,AB)</th>
<th>(AB,AA)</th>
<th>(AA,AA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>h h, h h</td>
<td>$\theta_{1,hh} \times \theta_{2,hh}$</td>
<td>$\bar{u}, \bar{u}$</td>
<td>$2u_A, \bar{u}$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
</tr>
<tr>
<td>h h, l h</td>
<td>$\theta_{1,hh} \times \theta_{2,hl}$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
</tr>
<tr>
<td>h h, l l</td>
<td>$\theta_{1,hh} \times \theta_{2,ll}$</td>
<td>$\bar{u}, \frac{1}{2}\bar{u}$</td>
<td>$2u_A, \frac{1}{2}\bar{u}$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
</tr>
<tr>
<td>l h, h h</td>
<td>$\theta_{1,hl} \times \theta_{2,hh}$</td>
<td>$u_A, \bar{u}$</td>
<td>$u_A, \bar{u}$</td>
<td>$u_A, 2u_A$</td>
<td>$u_A, 2u_A$</td>
</tr>
<tr>
<td>l h, l h</td>
<td>$\theta_{1,hl} \times \theta_{2,hl}$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
</tr>
<tr>
<td>l h, l l</td>
<td>$\theta_{1,hl} \times \theta_{2,ll}$</td>
<td>$\bar{u}, \frac{1}{2}\bar{u}$</td>
<td>$2u_A, \frac{1}{2}\bar{u}$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
</tr>
<tr>
<td>l l, h h</td>
<td>$\theta_{1,hl} \times \theta_{2,hh}$</td>
<td>$\frac{1}{2}\bar{u}, \bar{u}$</td>
<td>$u_A, \bar{u}$</td>
<td>$\frac{1}{2}\bar{u}, 2u_A$</td>
<td>$u_A, 2u_A$</td>
</tr>
<tr>
<td>l l, l h</td>
<td>$\theta_{1,hl} \times \theta_{2,hl}$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
</tr>
<tr>
<td>l l, l l</td>
<td>$\theta_{1,hl} \times \theta_{2,ll}$</td>
<td>$\bar{u}, \frac{1}{2}\bar{u}$</td>
<td>$2u_A, \frac{1}{2}\bar{u}$</td>
<td>$\bar{u}, u_A$</td>
<td>$2u_A, u_A$</td>
</tr>
</tbody>
</table>

Expected payoff for the case where (*) does not hold. Note: $\bar{u} = u_A + u_B$

For (AA): $\theta_{1,hh} = \theta_A^2$ $\theta_{1,hl} = \theta_A(1 - \theta_A)$ $\theta_{1,ll} = (1 - \theta_A)^2$

For (AB): $\theta_{1,hh} = \theta_A^2\theta_B$ $\theta_{1,hl} = \theta_A(1 - \theta_B)$ $\theta_{1,ll} = (1 - \theta_A)\theta_B$ $\theta_{1,ll} = (1 - \theta_A)(1 - \theta_B)$

The ex ante expected payoff for (Not Disclose, Not Disclose) is the weighted sum of the above expected payoff condition on test score realization, with the weights the probability of the test score realization.