Midterm Exam 1, Friday, September 29, 2017

Instructions: Do all six numbered problems. If you wish, you may also attempt the two optional bonus questions. Show all work, including scratch work. Little or no credit may be awarded, even when your answer is correct, if you fail to follow instructions for a problem or fail to justify your answer. Simplify your answers whenever possible, and draw your graphs large and clearly labelled. If you need more space, use the back of any page. If you have time, check your answers.

WRITE LEGIBLY. NO CALCULATORS.

1. (15 points) Find an equation for the plane that contains the points \( P = (1, 0, 1), \) \( Q = (2, 2, 0), \) and \( R = (2, 1, -1). \)

2. (15 points) Let \( \vec{a} = \langle 2, -1, 3 \rangle \) and \( \vec{b} = \langle 1, -2, 1 \rangle. \)

   (2a) Let \( \theta \) be the angle between \( \vec{a} \) and \( \vec{b}. \) Compute either \( \cos \theta \) or \( \sin \theta. \) (Your choice, but specify which one you are computing).

   (2b) Compute the (vector) projection \( \text{proj}_{\vec{b}} \vec{a} \) of \( \vec{a} \) onto \( \vec{b}. \)

3. (15 points) Let \( L_1 \) be the line given by \( \vec{r}_1(t) = \langle t + 2, 3t + 4, 2t + 5 \rangle, \) and let \( L_2 \) be the line given by \( \vec{r}_2(t) = \langle 3 - 2t, 1 - 3t, t - 3 \rangle. \)

   (3a) Show that \( L_1 \) and \( L_2 \) intersect, and find their point of intersection.

   (3b) Find an equation for the line parallel to \( L_2 \) that passes through the point \( (1, 4, -2). \)

4. (30 points) Sketch the graphs of the following two quadric surfaces. Provide either some “trace” curves in separate graphs, or some brief verbal description, or both, to help explain what your graph is trying to show, and to explain how you obtained it.

   (4a) \( 4x^2 + y + z^2 = 0 \)

   (4b) \( -x^2 + 4y^2 - z^2 = 4 \)

5. (10 points) Write out a definite integral whose value is the length of the curve given by \( \vec{r}(t) = \langle \sqrt{t + 1}, \cos(\pi t), t^2 \rangle \) from the point \( (1, 1, 0) \) to the point \( (2, -1, 9). \)

   Do not evaluate the integral.

6. (15 points) Let \( \vec{r}(t) = \langle \sin(3t), 2t, \cos(3t) \rangle. \)

   (6a) Compute \( \vec{r}(0) \) and \( \vec{r}'(0). \)

   (6b) Graph the curve traced out by \( \vec{r}(t) \). On your graph, mark the point \( \vec{r}(0), \) with the vector \( \vec{r}'(0) \) emanating from it.

OPTIONAL BONUS A. (2 points) Suppose a particle is moving through space, with position at time \( t \) given by some vector-valued function \( \vec{r}(t). \)

Suppose also that at all times \( t, \) the velocity vector \( \vec{r}'(t) \) is orthogonal to the acceleration vector \( \vec{r}''(t). \) Prove that the speed \( \|\vec{r}'(t)\| \) of the particle is constant.

OPTIONAL BONUS B. (1 point) On Monday, an independence referendum was held in a certain region of a certain country. Voters in the region overwhelmingly voted for independence, but the central government of the country is not recognizing the legitimacy of the referendum, and the governments of several neighboring nations and of the US have denounced the vote. Name the country in question; and also name either the region that held the vote, or else the dominant ethnic group in that region.