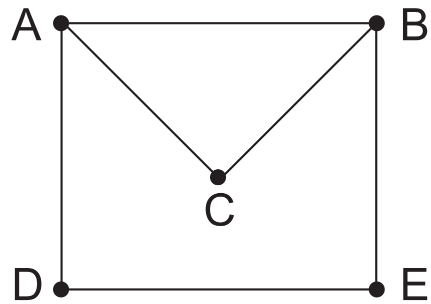
# Math 365 Simple Random Walk on a Graph

The objective of this lab is to study a random walk on a small graph through a combination of simulations and analysis using R. Please email your R script with answers to the exercises included as comments to tleise@amherst.edu.

## 1 Problem set-up

Consider the simple random walk on the graph shown below. We define a Markov chain which moves from vertex to vertex by randomly following an edge to an adjacent vertex. For example, $\mathbb{P}\{X_{n+1} = A | X_n = D\} = 1/2$, $\mathbb{P}\{X_{n+1} = C | X_n = A\} = 1/3$ and $\mathbb{P}\{X_{n+1} = B | X_n = D\} = 0$.




Determine the transition matrix $\mathbf{P}$ for this Markov chain with states $\{A, B, C, D, E\}$.

## 2 Analysis of the Markov chain

1. Calculate the invariant probability vector $\bar{\pi}$. In the long run, about what fraction of the time is spent at vertex $A$?

2. Suppose the random walk begins at vertex $A$. What is the expected number of steps until the walk returns to $A$?

   We can use simulations to generate the distribution of return times, as the expected value doesn't tell the full story. First define a function that can randomly step from vertex to vertex:

```
takestep <- function(x) {
switch(x,
sample(c(2,3,4),1), #A
sample(c(1,3,5),1), #B
sample(c(1,2),1), #C
sample(c(1,5),1), #D
sample(c(2,4),1)) #E
}
```

   Next define a function that finds the return time to a desired vertex for each simulation:

```
waitingtime <- function(vertex) {
x <- vertex
for (j in 1:10000) {
x <- takestep(x)
ifelse(x==vertex, return(j),-1) # takes steps until hits vertex again
}  }
```

Run many simulations and plot a histogram of the return times:

```
nsims <- 100000
sims <- matrix(0,nsims)
for (k in 1:nsims) sims[k] <- waitingtime(1) # vertex A
hist(sims,(min(sims)-1):(max(sims)+1),freq=FALSE,
xlab="Number of steps",xlim=c(0,20),
main=paste("Mean first return time is",mean(sims)))
```

3. Suppose the random walk begins at vertex $C$. What is the expected number of steps until the walker reaches $A$? Hint: Make $A$ an absorbing state, then calculate matrices $\mathbf{Q}$ and $\mathbf{M}$.

4. Suppose the random walk begins at vertex $C$. What is the expected number of visits to B before the walker reaches $A$?

5. Suppose the random walk begins at vertex $B$. What is the probability that the walker reaches $A$ before $C$?