Practice Problems for Math 320 Exam 2

The exam will be in class on Monday, December 7, and will cover Chapters 7-9.

Key concepts include:

1. Fourier transform and its properties (all those on the handout)
2. Riemann-Lebesgue Lemma
3. Shannon Sampling Theorem (note there is a typo in the book's statement on p210, so use version from lecture or lab)
4. Uncertainty Principle
5. Haar system
6. Expectation and difference operators

Practice problems (exam will have problems similar to these):

Problem 1 State the Shannon Sampling Theorem.

Problem 2 State the Uncertainty Principle and briefly explain its implications for time localization versus frequency localization.

Problem 3 Prove the following property of Fourier transform (assume a nice function $f(x)$ like a Schwartz function): if $g(x) = f(x-a)$ for some $a \in \mathbb{R}$, then $\hat{g}(\omega) = e^{-2\pi i \omega a}\hat{f}(\omega)$.

Problem 4 Prove the following property of Fourier transform (assume a nice function $f(x)$ like a Schwartz function): if $g(x) = f(ax)$ for some $a \neq 0$, then $\hat{g}(\omega) = \frac{1}{|a|}\hat{f}\left(\frac{\omega}{a}\right)$.

Problem 5 Prove the following property of Fourier transform (assume a nice function $f(x)$ like a Schwartz function): if $g(x) = e^{2\pi i a x} f(x)$ for some $a \in \mathbb{R}$, then $\hat{g}(\omega) = \hat{f}(\omega - a)$.

Problem 6 Prove that the $L^2(\mathbb{R})$ norm of a Schwartz function $f(x)$ equals the norm of its Fourier transform.

Problem 7 What is the Fourier transform of the convolution of two functions $f, g \in S(\mathbb{R})$? Prove this result.

Problem 8 Find $P_0 f$(x) and $P_1 f(x)$ of $f(x) = 2x$ on $[0,1)$, zero otherwise. Use these to determine $Q_0 f(x)$ in terms of Haar functions $h_{0,k}(x)$.

Problem 9 Find $P_1 f$(x) and $P_2 f(x)$ of $f(x) = 4$ on $[-1, 3/4]$, zero otherwise. Use these to determine $Q_1 f(x)$ in terms of Haar functions $h_{1,k}(x)$.

Problem 10 State the Riemann-Lebesgue Lemma for $f \in S(\mathbb{R})$.

Problem 11 What function equals its own Fourier transform?