Matlab and Fourier Series (Due Wed 9/23)

The objectives of this lab include plotting partial sums of Fourier series, calculating how many terms are needed to approximate a function within a desired accuracy, and examining the Gibbs phenomenon.

We will use the trigonometric form of Fourier series in this lab, rather than the complex exponential form. In particular we will focus on the sine series, which can be used to represent odd functions (or odd extensions of a function on a given interval, with period twice the length of the interval). Suppose we want to approximate a function $f(t)$ on the interval $[0, \pi]$. The idea is to extend $f(t)$ by defining $f(t) = -f(-t)$ on $[-\pi, 0]$, and then periodically extending this function on $[-\pi, \pi]$ with period 2π to the entire real line. This leads to the following formula for the nth coefficient of the corresponding sine series:

$$b_k = \frac{2}{\pi} \int_0^\pi f(t) \sin(kt) dt, \tag{1}$$

where

$$f(t) \sim \sum_{k=1}^{\infty} b_k \sin(kt).$$

The use of $\sim$ is to remind us that this is a representation of the function f that may agree with the function at some points but doesn't necessarily converge to $f(t)$ at all points.

The nth partial sum S_n of the sine series is defined by

$$S_n(t) = \sum_{k=1}^{n} b_k \sin(kt).$$

Instructions: Submit the three requested figures as .fig or .tif files. You can save figures by clicking on File in the figure window, then choosing Save As and naming the file after the corresponding exercise, e.g., JonesLab1Exercise1.fig. When finished with all three exercises, email the three figure files as attachments to tleise@amherst.edu.

Exercise 1 Find the sine series coefficients for $f(t) = 1$ on $[0, \pi]$ by hand using equation (1), and then plot the nth partial sums of the sine series for $n = 10, 20, 50$ (all three graphs on the same set of axes). Use a legend to label the curves: `legend('n=10','n=20','n=50')`. Determine the largest overshoot of $S_n(t)$ on [0,1] to three decimal places for each n as well as where it occurs (closest to $t = 0$), and add this information to the legend, so labels look something like `'n=10, max overshoot 0.123 at t=0.123'`.

Hint on coding Matlab: You can use for-loops to generate the coefficients and the partial sums. For example,

```
b=zeros(50,1);
for k=1:50
 b(k)=fill in formula;
end
```

```
t=0:0.01:pi;
S50=zeros(1,length(t));
for k=1:50
 S50=S50+b(k)*sin(k*t);
end
plot(t,S50)
```

Here is a more parsimonious (but less obvious) approach to computing the partial sum:

```
S50=sum(repmat(b(1:50),1,length(t)).*sin((1:50)'*t))
```

The `max` command may also be helpful: `[m, k]=max(x)` gives the maximum value `m` in the vector `x` as well as the corresponding index `k`, so `t(k)` would be the time at which the maximum value occurred.

Exercise 2 The sine series coefficients for $f(t) = t(\pi - t)$ are

$$b_k = \frac{4(1-(-1)^k)}{k^3\pi}.$$

Plot the nth partial sum of the sine series for $n = 1, 3, 5$ on the same plot as the function itself. To plot the function $f(t)$, use `plot(t,t.*(pi-t))`. Use a legend to identify the curves. Determine the smallest value of n such that $|f(t) - S_n(t)| < 0.001$ for $0 \le t \le \pi$. State this value of n in the title to the figure: `title('n=# is required to achieve pointwise error < 0.001')`.

Exercise 3 To compare the decay rates of the Fourier coefficients of the discontinuous function in Exercise 1 and the continuous function in Exercise 2, plot the odd-numbered coefficients of each sine series divided by the first coefficient, with a legend. That is, plot b_k/b_1 for $k = 1, 3, 5, ..., 49$. (Note that the even-numbered coefficients of these two sine series happen to be zero, due to the symmetry of the functions.)