Math 211 Spring 2017 Final Exam **May 11, 2017**

You are not allowed to use books, notes or calculators. Explain your answers completely and clearly to receive full credit. Please turn off all electronic devices and anything else that could cause distractions.

1. (10 points) Find parametric equations for the curve of intersection of the surfaces given by $x^2 = 2y$ and $z = 3xy$, and then find parametric equations for the tangent line to that curve of intersection at the point (-2,2,-12).

2. (15 points) Consider the two lines described by $\mathbf{r_1}(t) = \langle 5t + 2,\ -4t,\ t - 7\rangle$ and $\mathbf{r_2}(t) = \langle 4 - 3t,\ 12 - t,\ -2t - 1\rangle$.

   (a) Find the point of intersection of these two lines.

   (b) Find the angle between these two lines.

   (c) Find an equation for the plane that contains both of these lines. Please simplify the equation to the form $ax + by + cz + d = 0$ and divide out any common factors.

3. (15 points) Consider the function

$$f(x,y) = \begin{cases} \dfrac{3x^2 - 4x^3 + 3y^2}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 3 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Compute $f_x(0,0)$ and $f_y(0,0)$.

   (b) Is $f(x,y)$ continuous at $(0,0)$? Explain.

4. (15 points) Let $f(x,y) = x \ln y + x$. Give all directions as unit vectors.

   (a) In what direction does f increase most rapidly if you are at the point (1,1)?

   (b) In what direction does f decrease most rapidly if you are at the point (1,1)?

   (c) In what directions does the directional derivative of f equal 0 if you are at the point (1,1)?

5. (10 points) Compute the triple integral $\iiint_E z dV$ where E is the region between the spheres $x^2 + y^2 + z^2 = 4$, $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{x^2 + y^2}$.

6. (10 points) Compute the line integral $\int_C y^2\, dx + 6xy\, dy$ where C is the boundary curve of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$, traversed in the counterclockwise direction.

7. (10 points) Use the Fundamental Theorem of Line Integrals to evaluate the line integral $\int_C \mathbf{F}(x, y, z) \cdot \mathbf{dr}$ where

$$\mathbf{F}(x, y, z) = \langle yze^{xyz},\ xze^{xyz},\ xye^{xyz} \rangle$$

and C is curve described by $\mathbf{r}(t) = \langle t^2,\ -t,\ t^3 \rangle$ for $0 \leq t \leq 1$.

8. (15 points) Find the absolute maximum and minimum values of the function $f(x, y) = xy - 1$ subject to the constraint $x^2 + y^2 \leq 2$. State all points where the extrema occur as well as the maximum and minimum values.

Extra Credit (5 points): Let $P(x, y)$ be a function defined on $\mathbb{R}^2$ with continuous partial derivatives. Let R be the rectangle $[a, b] \times [c, d]$ and C be its boundary traversed counterclockwise. Directly prove (without using Green's Theorem) that

$$\int_C P\, dx = -\iint_R \frac{\partial P}{\partial y}\, dA.$$