Fall 2012 Math 211 Exam 3 Review Sheet

The exam will be in class on Thursday, December 3, and will cover Sections 14.8 and 15.2-9. You will not be allowed use of a calculator or any other device other than your pencil or pen and some scratch paper. Notes are also not allowed. In kindness to your fellow test-takers, please turn off all cell phones and anything else that might beep or be a distraction.

Important topics:

- Optimization with a constraint (Lagrange multipliers)
- Iterated integrals (double and triple integrals)
- Finding mass and center of mass given a density function
- Finding average value of a function in a region (2D or 3D)
- Integrals in polar, cylindrical and spherical coordinates
- Surface area
- Basic integration rules like substitution (but you do not need to know integration by parts for the exam)

Review exercises from Chapter 14: 59,60,63 (answer to #60: max of $\sqrt{2}$, min of $-\sqrt{2}$)

Review exercises from Chapter 15: 5,9,13,17,21,25,27,31,33,39,41,42,47 (#42: $\frac{64\pi}{9}$)

If you feel confident in working the review problems under the exam rules, then you should be well prepared for the exam. If you have problems with certain topics or want more practice, feel free to stop by my office or send an email.

<u>Practice Exam Problems</u> (Exam will have 5 problems similar in difficulty to these)

1. The shape of a space probe entering the Earth's atmosphere is the ellipsoid $4x^2+y^2+4z^2=16$. The probe's surface is heated by re-entry so that at a particular time the temperature is given by $T(x,y,z)=8x^2+4yz-16z+600$. Find the hottest point(s) on the probe's surface.

2. Find the absolute maximum and minimum values of the function $f(x,y,z)=x-y+z$ subject to the constraint $x^2+y^2+z^2=1$.

3. Sketch the region of integration, then reverse the order of integration:

   a. $$\int_0^1 \int_{1-x}^{1-x^2} f(x,y)\,dy\,dx$$

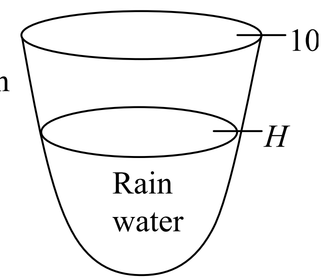
   b. $$\int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\,dx\,dy$$

4. Find the volume of the part of the sphere $x^2 + y^2 + z^2 = 2$ that lies inside the paraboloid $z = x^2 + y^2$.

5. Evaluate the triple integral $\iiint_E \sqrt{x^2 + y^2 + z^2}\,dV$, where E is the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ with $z \geq 0$ (above the xy-plane).

6. Evaluate $\iiint_E \left(x^2 + y^2 + z^2\right)^{5/2} dV$ where E is bounded by the xz-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$.

7. Suppose you want to calibrate a bowl shaped like a paraboloid ($z=x^2+y^2$ for $z=0$ to 10 inches) to be a rain gauge. For example, if it "rained 2 inches," then the volume of rain collected in the bowl would be 2 inches times the area of the opening, 10π in^2, and you would mark the resulting height of the water in the bowl as the "2 inches of rain" level. Derive a general formula that relates the number of inches of rain R to the resulting height H of water collected in the bowl.



8. Find the center of mass of the solid below the paraboloid $z = 2 - x^2 - y^2$ and above the cone $z = \sqrt{x^2 + y^2}$ if the density is constant.

9. Find the volume of the part of the sphere $x^2 + y^2 + z^2 = a^2$ that lies within the cylinder $x^2 + y^2 = ax$ and above the xy-plane, where $a>0$.

10. Evaluate the triple integral $\iiint_E z^2 dV$, where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$.

11. Find the volume of the solid region bounded by $y=0$, $y=3$, $z=x^2$, and $z=2-x^2$.

12. Use polar or cylindrical coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and above the cone $z = \sqrt{x^2 + y^2}$.

13. Use spherical coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and above the cone $z = \sqrt{x^2 + y^2}$.

14. Find the average value of $f(\rho,\phi,\theta) = \rho$ over the ball $\rho \leq 1$.

15. A 1-inch radius cylindrical hole is drilled through the center of a ball with 2-inch radius. Find the volume of the core that was removed from the ball.

16. Find the surface area of the ellipsiodal-shaped region cut from the plane $z=cx$ (c is a constant) by the cylinder $x^2+y^2=1$.

Answers:

1. Hottest points on surface are (4/3,-4/3,-4/3) and (-4/3,-4/3,-4/3).

2. Absolute max of √3 at (1,-1,1)/√3 and absolute min of -√3 at (-1,1,-1)/√3.

3. (a) $\int_0^1 \int_{1-y}^{\sqrt{1-y}} f(x,y)\,dx\,dy$ (b) $\int_1^2 \int_0^{\ln x} f(x,y)\,dy\,dx$

4. $\pi(8\sqrt{2}-7)/6$
5. $15\pi/2$

6. $\frac{\pi}{4}(4^8 - 3^8)$

7. $20R=H^2$
8. (0,0,11/10)
9. $(3\pi-4)a^3/9$

10. $124\pi/15$

11. $V = \int_0^3 \int_{-1}^1 (2 - x^2 - x^2)\,dx\,dy = 8$

12. $V = \int_0^{2\pi} \int_0^{\frac{3}{\sqrt{2}}} \int_r^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta = \int_0^{2\pi} \int_0^{\frac{3}{\sqrt{2}}} (\sqrt{9-r^2} - r) r\,dr\,d\theta = 9\pi(2-\sqrt{2})$

13. $V = \int_0^{2\pi} \int_0^{\pi/4} \int_0^3 \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta = 9\pi(2-\sqrt{2})$

14. $Average = \frac{1}{V} \int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^3 \sin\phi d\rho d\phi d\theta = \frac{\pi}{\frac{4}{3}\pi} = \frac{3}{4}$

15. $V = \int_0^{2\pi} \int_0^1 \left(\sqrt{4-r^2} - \left(-\sqrt{4-r^2}\right)\right) r dr d\theta = 4\pi\left(\frac{8}{3} - \sqrt{3}\right)$

16. $\pi\sqrt{1+c^2}$

Please let me know if you find any errors in these answers. Thanks!

Practice Exam (no solutions available)

1. (20pt) Evaluate the double integral $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\,dy\,dx$ and sketch the region of integration.

2. (20pt) A flat circular plate is bounded by $x^2 + y^2 = 4$. The plate (including the boundary) is heated so that the temperature is $T(x,y) = x^2 + 3y^2 + 6y + 12$ at each point (x,y) on the plate. Find the temperatures at the hottest and coldest points on the boundary $x^2 + y^2 = 4$ of the plate.

3. (20pt) Use **spherical coordinates** to find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

4. (20pt) Find the **surface area** of the part of the cone $z^2 = a^2(x^2 + y^2)$ between the planes z=1 and z=2, where a>0 is a constant.

5. (20pt) Use **polar coordinates** to find the average height of the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ above the disk $x^2 + y^2 \le a^2$, where a>0 is a constant. (You can easily check your answer using standard geometry formulas for area and volume, but for full credit, evaluate the answer using a double integral involving polar coordinates.)