HADRONS AS KERR-NEWMAN BLACK HOLES

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Abstract: The conformal invariance of the Einstein field equations allows one to model hadrons as "strong gravity" black holes if one uses appropriate scaling of units and a revised gravitational coupling factor. The inner consistency of this hypothesis is demonstrated by deriving the mass of the proton from an equation that relates the angular momentum and mass of a Kerr-Newman black hole. The radius of a helium nucleus is derived in a second test, and an alternative Planck scale is proposed.

I. Introduction

The Einstein field equations of General Relativity can be written[1] as:

$$R_{\mu\nu} - 1/2\, g_{\mu\nu}\, R = k\, T_{\mu\nu} \qquad (1)$$

where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, R is the Ricci scalar, $T_{\mu\nu}$ is the stress-energy tensor and k is the coupling factor between the geometry of a space-time and its matter content. This equation can be written in an even more compact form:

$$G_{\mu\nu} = k\, T_{\mu\nu} \qquad (2)$$

where $G_{\mu\nu}$ is called the Einstein tensor. This deceptively simple expression disguises the fact that the equation represents a complicated and coupled set of 10 nonlinear partial differential equations in 4 unknowns. However, the conceptual meaning of the equation has a simple elegance. The geometry of the space-time ($G_{\mu\nu}$) is determined by the energy and momentum densities/fluxes of the matter ($T_{\mu\nu}$) and, reciprocally, the motions of the matter are determined by the geometry of the space-time.

For the purposes of the present discussion, we need only focus on the term k in Eqs. (1) and (2). Einstein and those who have followed in his footsteps noted that if

$$k = [8\pi/c^4]\, G_N \qquad (3)$$

where G_N is the conventional Newtonian gravitational constant and c is the velocity of light, then General Relativity successfully predicts the observed advance of the perihelion of Mercury, the deflection of light rays passing near to the Sun, the observed frame-dragging of space-time, and all the usual macroscopic results approximated by Newtonian gravitation. However, in the late 1970s and early 1980s several physicists including A. Salam[2], C. Sivaram[3], K. P. Sinha[3], and E. Recami[4] explored the theoretical possibility of "strong gravity" within the microcosm.

II. "Strong Gravity"

The rationale for "strong gravity" is as follows. Both General Relativity and Maxwell's equations manifest conformal invariance if masses, charges and dimensional "constants" are suitably scaled. One can hypothesize a global discrete dilation invariance (a form of conformal invariance) wherein *all* length (L), time (T) and mass (M) *units* scale according to discrete transformations of the form:

$$L_N = \Lambda\, L_{N-1} \qquad (4)$$

$$T_N = \Lambda\, T_{N-1} \qquad (5)$$

$$M_N = \Lambda^D\, M_{N-1} \qquad (6)$$

where Λ and D are dimensionless scaling constants, and where N and N-1 designate two neighboring discrete scales with a hierarchical arrangement, i.e., macrocosm and microcosm. Going back to Eq. (2), we can consider the idea that, whereas k applies in the macrocosm,

$$k' = [8\pi/c^4][\, \Lambda^{D-1}\, G_N] = [8\pi/c^4]\, G_{N-1} \qquad (7)$$

applies in the subatomic realm. The term Λ^{D-1} arises because the dimensionality of G_N is L^3/MT^2, and therefore $G_{N-1} = \Lambda^{D+2-3}\, G_N$. For the case of hadrons, it is at least logically possible that the gravitational coupling between matter and the geometry of space-time is much stronger than for stellar systems.

One might well ask whether there are observational results or theoretical evidence that supports the "strong gravity" hypothesis. In fact there are some interesting data which are consistent with this unorthodox idea. As discussed in detail by Sivaram and Sinha[3], hadrons and Kerr-Newman black holes share an intriguing set of similarities.

1. Both hadrons and Kerr-Newman black holes are almost entirely characterized by just three parameters: mass, charge and angular momentum.
2. Both hadrons and Kerr-Newman black holes have magnetic dipole moments, but do <u>not</u> have electric dipole moments.
3. Typical hadrons and Kerr-Newman black holes have gyromagnetic ratios of ≈ 2.
4. Hadrons and Kerr-Newman black holes have similar linear relationships between angular momentum and mass squared, i.e., $J \alpha M^2$.
5. When Kerr-Newman black holes interact, their surface areas may increase but can never decrease; this is analogous to the increase of cross-sections in hadron collisions.

Given these curious similarities between the fundamental characteristics of hadrons and Kerr-Newman black holes, there is ample motivation for considering the "strong gravity" approach to hadrons.

III. Two Tests

What we would like are some empirical tests of the largely theoretical arguments for "strong gravity", and fortunately two appropriate observational tests have been devised and evaluated. These quantitative tests require a determination of G_{N-1} from Eq. (7). After analyzing a large and varied sample of comparative data from subatomic and stellar scale systems[5-7], the best empirical fit for

$$G_{N-1} = [\Lambda^{D-1} G_N] = 2.18 \times 10^{31} \text{ cm}^3/\text{g sec}^2 \qquad (8)$$

where $\Lambda = 5.2 \times 10^{17}$ and $D = 3.174$. The values for these dimensionless constants were determined *decades before the present tests were considered*, using a *different set of observational data*.[6]

For a Kerr-Newman black hole there is an angular momentum (J) versus mass (M) relationship[8] of the form:

$$J = aG_N M^2/c \qquad (9)$$

where a is the dimensionless spin of the black hole and c is the velocity of light. If the "strong gravity" hypothesis has merit, then we should be able to apply Eq. (9) to a stable hadron and get reasonable empirical results. Taking the proton as the archetypal "strong gravity" analogue for a macroscopic Kerr-Newman black hole, we have

$$\hbar = (1/2)\, G_{N-1}\, m^2/c \qquad (10)$$

where ħ is Planck's constant divided by 2π, m is the proton mass and the dimensionless spin parameter is ½. Eq. (10) can be rearranged and reduced to

$$m = [hc/\pi\ G_{N-1}]^{1/2}\ . \qquad (11)$$

When we evaluate Eq. (11) for m, we get

$$m = [(6.63 \times 10^{-27} \text{ erg sec})(2.99 \times 10^{10} \text{ cm/sec})/\pi(2.18 \times 10^{31} \text{ cm}^3\text{/g sec}^2)]^{1/2}$$

$$= 1.70 \times 10^{-24} \text{ g}\ .$$

Our estimate for m based on the "strong gravity" hypothesis is in agreement with the measured value of 1.67×10^{-24} g at the 98.3% level. For atomic nuclei with half-integer spin ($a > 0$), we have the general relation

$$m_A \approx A[\hbar c/aG_{N-1}]^{1/2} \qquad (12)$$

where A is the rounded-off Atomic Mass Units, i.e., the number of nucleons.

A second test involves a radius calculation, rather that a mass calculation. The alpha particle has a spin of 0 and therefore we cannot use the Kerr-Newman solution of General Relativity for this test. However, we can use a Schwarzschild solution for an order-of-magnitude check on the appropriateness of G_{N-1} for a system that is more massive than the proton and has $a = 0$. For the Schwarzschild metric in the stellar scale context

$$R = 2\ G_N M/c^2 \qquad (13)$$

where R is the radius of a Schwarzschild black hole. In the case of an alpha particle we hypothesize that

$$r_\alpha \approx 2G_{N-1}m_\alpha/c^2 \qquad (14)$$

where r_α and m_α are the radius and mass of the alpha particle, respectively. Evaluating Eq. (14), we get

$$r_{\alpha,Sch} \approx (2)(2.18 \times 10^{31} \text{ cm}^3\text{/g sec}^2)(6.68 \times 10^{-24} \text{ g})/(2.99 \times 10^{10} \text{ cm/sec})^2$$

$\approx 3.26 \times 10^{-13}$ cm.

The measured value for $r_{\alpha,emp} \approx 2.2 \times 10^{-13}$ cm, and therefore $r_{\alpha,Sch} \approx 1.5\, r_{\alpha,emp}$. Given that the Schwarzschild solution probably offers only a rough approximation for any actual physical system in nature, we have achieved a reasonable level of agreement between the observed radius of the alpha particle and an order-of-magnitude theoretical estimate based on the "strong gravity" hypothesis.

IV. Conclusions

Given the fact that G_{N-1} is evaluated empirically and is thus an approximation, the general agreement between the empirical and theoretical values for the proton mass and the alpha particle radius encourages one to think that the hypothesis of "strong gravity" is worth pursuing. A follow-up[9] to the present research suggests that the proposed revision of the k term in General Relativity leads to a radical reconsideration of assumptions relating to the Planck Scale, which is the microcosmic scale at which General Relativity and Quantum Electrodynamics play equally important dynamical roles. If G_{N-1} is used in place of G_N when one calculates the Planck length, Planck mass and Planck time, the results are as follows.

Planck length $\equiv (\hbar\, G_{N-1}/c^3)^{1/2} = 2.9 \times 10^{-14}$ cm ($\approx 0.4\, r_{proton}$).

Planck mass $\equiv (\hbar c/G_{N-1})^{1/2} = 1.2 \times 10^{-24}$ g ($\approx 0.7\, m_{proton}$).

Planck time $\equiv (\hbar\, G_{N-1}/c^5)^{1/2} = 9.8 \times 10^{-25}$ sec ($\approx 0.4\, r_{proton}/c$).

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