Math 211, Section 01, Fall 2017

Practice Problems for Midterm Exam 1

(A little harder than, and two to three times as long as, the real exam)

1. Find an equation for the plane that passes through the point (1, 2, 2) and contains the line \( \mathbf{r}(t) = 2t\mathbf{i} + (3 - t)\mathbf{j} + (1 + 3t)\mathbf{k} \).

2. Find an equation for the plane containing the three points (1, −2, 4), (2, 1, 1), and (−4, 0, −1).

3(a). Find an equation for the line of intersection of the planes \( x - 2y + z = 1 \) and \( 2x - y - z = 5 \).

(b). Find the cosine of the angle between these two planes.

4. Find an equation for the plane that contains the line in problem 3(a) above and also contains the point (1, 0, −2).

5. Let \( \mathbf{a} = 5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} \) and \( \mathbf{b} = \mathbf{i} - 3\mathbf{k} \). Find the (vector) projection of \( \mathbf{a} \) onto \( \mathbf{b} \), and find the (vector) projection of \( \mathbf{b} \) onto \( \mathbf{a} \).

6. Let \( P_1 = (1, 1, −1) \), \( P_2 = (3, −2, 0) \), \( P_3 = (2, −3, 0) \), and \( P_4 = (4, −6, 1) \).

(a). Confirm that the four points \( P_1, P_2, P_3, P_4 \) form the vertices of a parallelogram.

(b). Find the area of the parallelogram with vertices \( P_1, P_2, P_3, P_4 \).

7. For each of the following pairs of lines, decide whether they cross, are parallel, or are skew. If they cross, find their point of intersection and the cosine of the angle between them.

(a). \( \mathbf{r}_1(t) = (1 + 3t, 3 - t, t) \) and \( \mathbf{r}_2(t) = (t + 4, 2t - 5, 5 - t) \)

(b). \( \mathbf{r}_1(t) = (1 - 2t, 3t + 1, t - 2) \) and \( \mathbf{r}_2(t) = (1 + t, -2t, 5t + 2) \)

(c). \( \mathbf{r}_1(t) = (3 + 2t, 4t - 7, 5 - 6t) \) and \( \mathbf{r}_2(t) = (2 - 3t, 1 - 6t, 9t + 4) \)

(d). \( \mathbf{r}_1(t) = (5 - t, 4 - 3t, 2t - 1) \) and \( \mathbf{r}_2(t) = (t + 2, 3t - 5, 5 - 2t) \)

(e). \( \mathbf{r}_1(t) = (5 - t, 4 - 3t, 2t - 1) \) and \( \mathbf{r}_2(t) = (4t - 1, t - 5, 5 - t) \)

8. Sketch graphs of the following surfaces. Show how you got to your answer (using trace graphs, for example), and draw your final graph large and clear.

(a). \( 4x^2 + y^2 + 25z^2 = 100 \)  (b). \( x^2 + 4y^2 - z^2 = 4 \)  (c). \( z = 4x^2 + y^2 \)

(d). \( 3x^2 - z^2 = 9 \)  (e). \( z = 4x^2 - y^2 \)  (f). \( z^2 = 4x^2 - y^2 \)

(g). \( y = 4z^2 + x^2 \)  (h). \( y = 4z^2 - x^2 \)  (i). \( x^2 - 4y^2 + z^2 = -4 \)

9. (a) Find a vector-valued function tracing out the curve of intersection of the cylinder \( x^2 + z^2 = 9 \) and the plane \( x + y + z = 1 \).

(b). Find the tangent line to the curve in part (a) at the point (3, −2, 0).

(over)
10. Let \( \mathbf{r}(t) = (1 - 2t, \frac{1}{t}, 2\ln t) \).

(a). Find the domain of \( \mathbf{r} \), and compute the derivative \( \mathbf{r}'(t) \) and speed \( \|\mathbf{r}'(t)\| \) (for any \( t \) in that domain).

(b). Compute the arclength of the curve from \( t = 1 \) to \( t = 3 \).

11. Sketch the curve traced out by each of the following vector-valued functions, and indicate with an arrow the direction in which the curve is traced. Provide some brief extra description of the curve, either by labelling points, giving a brief but precise verbal description of it, or by an alternate mathematical equation for the curve, or some combination of such descriptions. Note: you may find it helpful in \( \mathbb{R}^2 \) to find a relationship between the \( x \) - and \( y \)-coordinates; and in \( \mathbb{R}^3 \) it may be helpful to find a surface that the curve lies in.

(a). \( \mathbf{r}(t) = (e^t, e^{2t} - 1) \), for \( t \in \mathbb{R} \).

(b). \( \mathbf{r}(t) = (2t, 4\sin(5t), \cos(5t)) \), for \( t \in \mathbb{R} \).

(c). \( \mathbf{r}(t) = (t \cos t, t \sin t, t^3) \), for \( t \geq 0 \).

12. For each of the following vector-valued functions, write down, but do not evaluate, a definite integral giving the arclength of the curve parametrized by the function.

(a). \( \mathbf{r}_1(t) = (t^2, e^{3t}, 2t + 3) \) for \( 0 \leq t \leq 4 \).

(b). \( \mathbf{r}_2(t) = (t \sin t, t \cos t, 2t) \) for \( -2 \leq t \leq 3 \).

(c). \( \mathbf{r}_3(t) = (3 \cos t, 2 \sin t) \).