LETTERS TO THE EDITOR

THE SPIN-STATISTICS THEOREM

In a 1994 "question" in this journal, Neuenschwander[1] asked whether anyone had yet met Feynman's challenge of providing an *elementary* proof of the spin-statistics theorem. This innocent question led to a series of supposed answers, then a "meta-answer" by Hilborn,[2] who pointed out that several earlier answers were perhaps interesting but in fact not truly relevant to the theorem in question, a long article by Duck and Sudarshan,[3] and a book[4] by the same authors. Although Duck and Sudarshan had hoped to find a proof that would satisfy Neuenschwander, they concluded, regretfully, that the best they could provide was an argument that was "still not completely free from the complications of relativistic quantum field theory."[5]

Recently, however, there was published in this journal a letter to the editor by Broyles,[6] in which it was claimed that a proof meeting Neuenschwander's criteria had been published long ago. This letter gave no supporting details but merely referred the reader to an unpublished 1999 posting on the Los Alamos e-print archive, a posting that primarily serves to point to a 1976 paper by Broyles himself.[7] Contrary to what one might infer from Broyles's recent letter, AJP authors Hilborn, Duck and Sudarshan (as well as the editors and referees who worked on the Duck and Sudarshan manuscript) were well aware of Broyles's 1976 paper and did not find it convincing. Lest readers be left with the impression that a definitive affirmative answer to Neuenschwander's question was given a quarter of a century ago, it seems important to point out that there are those who do not agree. To quote Duck and Sudarshan (Ref. 3, p. 291): "We conclude that Broyles' Postulate A is *ad hoc* special pleading which has no other purpose than to construct his proof of the Spin-Statistics Theorem, plays no other role, and is to be deleted from the lexicon of quantum mechanics." And Hilborn (Ref. 2) writes: "[Broyles's] argument establishes a spin-statistics connection at the expense of an *additional assumption*..." (italics added).

In spite of the importance of the spin-statistics theorem and the attention that has been devoted to it, the physics community still waits—probably in vain—for an *elementary* proof.[8] We are all indebted to Neuenschwander for reminding us of Feynman's challenge; though the direct answer to Neuenschwander's question is still negative, publication of his question has led to a better understanding of the theorem and its implications for those of us who have followed with interest the recent discussion.[9]

[1] Dwight E. Neuenschwander, "Question #7. The spin-statistics theorem," Am. J. Phys. **62**(11), 972 (1994).

[2] Robert C. Hilborn, "Answer to Question #7," Am. J. Phys. **63**(4), 298–299 (1995).

[3] Ian Duck and E. C. G. Sudarshan, "Toward an understanding of the spin-statistics theorem," Am. J. Phys. **66**(4), 284–303 (1998). See also E. C. G. Sudarshan, in *Spin-Statistics Connection and Commutation Relations*, edited by Robert C. Hilborn and Guglielmo M. Tino (American Institute of Physics, Melville, NY, 2000), pp. 40–54.

[4] Ian Duck and E. C. G. Sudarshan, *Pauli and the Spin-Statistics Theorem* (World Scientific, Singapore, 1997). See also the review of this book by Arthur Wightman [Am. J. Phys. **67**(8), 742–746 (1999)], and a summary of these various items in the June 2001 issue of AJP [Robert H. Romer, "Editor's choice," Am. J. Phys. **69**(6), 635–647 (2001)], especially pp. 635 and 641.

[5] Reference 4, p. 485. See also Ref. 3, p. 300 (referring to a 1975 argument by Sudarshan): "Sudarshan's proof eliminates the explicit dependence of the proof on relativistic quantum field theory. A critical implicit dependence on relativity is still present, however."

[6] Arthur A. Broyles, "Spin-statistics theorem," Am. J. Phys. **70**(2), 103 (2002).

[7] Arthur A. Broyles, "Spin and statistics," Am. J. Phys. **44**(4), 340–343 (1976).

[8] As one further note, Berry and Robbins have recently published an interesting article on the subject [M. V. Berry and J. M. Robbins, "Indistinguishability for quantum particles: spin, statistics and the geometric phase," Proc. R. Soc. London, Ser. A **453**, 1771–1790 (1997)]. See also Berry and Robbins in Hilborn and Tino (Ref. 3, pp. 3–15). The Berry and Robbins argument surely does not merit the adjective "elementary"; further, their argument seems potentially vulnerable to the same sorts of criticisms as those directed by Hilborn (Ref. 2) at the initial AJP "answers".

[9] What Neuenschwander actually asked was whether anyone had made any *progress* toward an elementary argument for the spin-statistics theorem (italics added). To *that* question, the various items discussed in this letter might be said to provide a cautiously positive answer. I might also add that many physicists are actually not quite clear on what the theorem *is*. Those who write most carefully on the subject are careful to draw a distinction between the spin-statistics *theorem* and the spin-statistics *connection*. Duck and Sudarshan write (Ref. 3, p. 301): "Although the Spin-Statistics Theorem is simply stated, it is by no means simply understood or simply proved." Hilborn would probably disagree with the "simply stated" portion of that remark or would at least point out that many of us harbor misconceptions as to what the theorem states. He writes (Ref. 2): "Many physicists believe that the infamous spin-statistics theorem of relativistic quantum field theory provides the theoretical basis for the spin connection. However, the theorem actually proves less than we would like, and it is disappointing in both its limitations and its lack of a physical picture of why the connection obtains. The theorem is actually a negative statement. It tells us what quantum field theory cannot do. ..."

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