

Aharonov-Casher-Effect Suppression of Macroscopic Tunneling of Magnetic Flux

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We suggest a system in which the amplitude of macroscopic flux tunneling can be modulated via the Aharonov-Casher effect. The system is an rf SQUID with the Josephson junction replaced by a Bloch transistor—two junctions separated by a small superconducting island on which the charge can be induced by an external gate voltage. When the Josephson coupling energies of the junctions are equal and the induced charge is $q = e$, destructive interference between tunneling paths brings the flux tunneling rate to zero. The device may also be useful as a qubit for quantum computation.

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Geometric-phase effects are ubiquitous in physics [1], the best known being the Aharonov-Bohm effect [2] in which charges moving in a field-free region surrounding a magnetic flux nevertheless pick up a phase that leads to interference. This effect long predates the general formulation of the geometric phase by Berry [3]. A related effect was proposed by Aharonov and Casher [4], who showed that a magnetic moment that moves around a line charge acquires a phase proportional to the linear charge density. Extending this analysis to superconductors, Reznik and Aharonov [5] showed that magnetic vortices (or fluxons) moving around an external charge q could acquire such a phase. Other theoretical work has concerned the appearance of this effect in Josephson-junction arrays and devices [6–11]. Experimental evidence for the Aharonov-Casher effect in a Josephson system was first provided by Elion *et al.* [12]. There vortices move around the charge induced on a superconducting island by an external gate voltage. Manifestations of the Aharonov-Casher effect in Josephson-junction systems studied up to now— $2e$ -periodic modulation of an array resistance with the induced charge and quantization of the voltage across the array—are, however, not specific to this effect. Qualitatively similar phenomena can be produced simply by the Coulomb-blockade effect due to the quantization of charge on the superconducting islands of the devices [13]. In this Letter, we suggest a system where the Aharonov-Casher effect manifests itself in a way that can be directly distinguished from Coulomb-blockade oscillations.

The proposed system (Fig. 1) is a variant of a SQUID. Macroscopic quantum tunneling of the magnetic flux in SQUID systems has been the subject of extensive experimental investigation in the past several years [14,15]. Recent experiments have shown that a SQUID can be put into a coherent superposition of macroscopically distinct flux [15] and persistent-current [16] states. The system we consider is a SQUID with the Josephson junction replaced by a Bloch transistor (see, e.g., [13])—two small, closely spaced Josephson junctions, in which the charge on the island between the junctions can be varied by an external gate voltage V_g . Because of the gate-voltage dependence

of the critical current of the Bloch transistor, the flux-tunneling rate in such a SQUID should exhibit $2e$ -periodic oscillations as a function of the gate-voltage-induced charge $q = C_g V_g$. As we show below, in addition to these (“Coulomb-blockade”) oscillations, the flux tunneling is completely suppressed when the induced charge is e . This fact does not follow from the simple charge quantization on the middle electrode of the Bloch transistor, but can be explained naturally in terms of the Aharonov-Casher effect.

In a handwaving sense, flux tunneling occurs via the passage of a fluxon through one of the junctions and interference can occur between the two paths that encircle the induced charge q on the middle electrode. When $q = e$, the relative phase of the two paths is π and the interference is destructive. If, in addition, the Josephson coupling energies E_{J1} and E_{J2} of the two junctions (which control the amplitude of flux tunneling through each individual junction) are equal, $E_{J1} = E_{J2}$ [17], the destructive interference is complete, and the total amplitude of flux tunneling is exactly zero. More generally, the Aharonov-Casher effect in this system makes it possible to control the phase of the flux-tunneling amplitude, an effect that can simplify design of Josephson-junction qubits for quantum computing [15,16,18–21].

Quantitatively, we consider a SQUID of inductance L (Fig. 1) that is small enough that the SQUID has only two metastable flux states [22]. The two junctions have

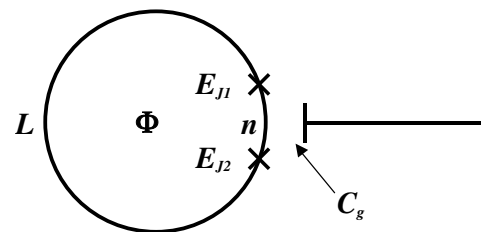


FIG. 1. Schematic of the proposed device, an rf-SQUID with the single Josephson junction replaced by a Bloch transistor. The charge on the superconducting island of the transistor can be induced by a gate voltage.

Josephson phase differences ϕ_1 and ϕ_2 across them. A gate electrode couples with capacitance C_g to the island between the junctions and an external flux Φ_x is applied to the SQUID loop. The Hamiltonian for this system is

$$H = \frac{Q^2}{2C} + \frac{(2en - q)^2}{2C_\Sigma} + \frac{(\Phi - \Phi_x)^2}{2L} - E_{J1} \cos\phi_1 - E_{J2} \cos\phi_2, \quad (1)$$

where n is the number of Cooper pairs charging the island, and C_Σ is its total capacitance relative to all other electrodes. The flux Φ in the SQUID is related to the phase differences across the junctions by $2\pi\Phi/\Phi_0 = \phi_1 + \phi_2$, where $\Phi_0 = h/2e$ is the flux quantum. Φ is conjugate to the charge Q on the capacitance C between the ends of the SQUID loop: $[\Phi, Q] = i\hbar$. (Because of the stray-capacitance contribution to C , it is not related directly to the island capacitance C_Σ .) Similarly, the phase $\theta = (\phi_1 - \phi_2)/2$ is conjugate to n : $[\theta, n] = i$. Thus, Φ and θ are the ‘‘coordinates’’ of the SQUID and Q and n are the ‘‘momenta.’’

The first two terms in the Hamiltonian represent the kinetic energy of the system and the remaining terms represent the potential energy, shown in Fig. 2 for the case $\Phi_x = \Phi_0/2$ and $E_{J1} = E_{J2}$. The potential has minima in a checkerboard-like pattern with minima on one side of $\Phi = \Phi_0/2$ shifted by π in the θ direction relative to minima on the other side. The minima are not separated by integer multiples of Φ_0 in the Φ direction because of the inductive term in Eq. (1). However, the potential is strictly periodic in the θ direction and, consequently, neighboring minima are always separated by $\Delta\theta = \pm\pi$. In fact, since the number n of Cooper pairs on the island is integer, one can impose periodic boundary conditions on θ , wrapping all points with $\theta = \pi$ to $\theta = -\pi$.

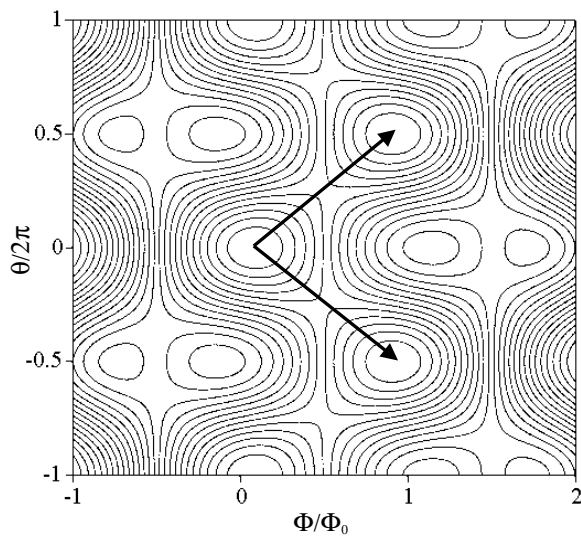


FIG. 2. Two-dimensional potential for the system described by Eq. (1) with $E_{J1} = E_{J2} = 0.507\Phi_0^2/2L$. When $q = e$, destructive interference between the two paths shown leads to suppression of flux tunneling.

We analyze tunneling between different flux states in two limits, one in which the charging energy for the island $E_C = (2e)^2/2C_\Sigma$ is much smaller than E_J and the other in which $E_C \gg E_J$. First, we consider the former case, where θ is a good quantum number and one can employ a tight-binding picture in which the system is localized in one of the minima of Fig. 2. The tunnel splitting can be found using an instanton technique in which the imaginary-time action S_I is evaluated along the least-action paths in the inverted potential [23]. The zero-temperature tunnel splitting [24] is then given by

$$\Delta = \sum_j \omega_j e^{-S_I^j/\hbar}, \quad (2)$$

where the sum is over all least-action paths between two potential minima, and ω_j and S_I^j are the attempt frequency and imaginary-time action, respectively, for the j th path. The action is calculated by evaluating

$$S_I^j = \int_{-\infty}^{\infty} \mathcal{L}_E(\tau) d\tau \quad (3)$$

along the j th path, where $\mathcal{L}_E(\tau) \equiv -\mathcal{L}(t \rightarrow -i\tau)$ is the imaginary-time (Euclidean) Lagrangian obtained from the real-time Lagrangian \mathcal{L} , which in turn is obtained from the Hamiltonian by the usual transformation: $\mathcal{L} = \dot{\Phi}Q + (\frac{\Phi_0}{2\pi}\dot{\theta})(2en) - H$. For $E_{J1} = E_{J2} = E_J$ and using Hamilton's equations to obtain $\dot{\Phi} = Q/C$ and $\frac{\Phi_0}{2\pi}\dot{\theta} = (2en - q)/C_\Sigma$ we find

$$\mathcal{L}_E(\tau) = \frac{C}{2} \dot{\Phi}^2 + \frac{C_\Sigma}{2} \left(\frac{\Phi_0 \dot{\theta}}{2\pi} \right)^2 + \frac{(\Phi - \Phi_x)^2}{2L} - 2E_J \cos(\pi\Phi/\Phi_0) \cos\theta - iq \left(\frac{\Phi_0 \dot{\theta}}{2\pi} \right), \quad (4)$$

where the dot represents differentiation with respect to τ . The last term in Eq. (4), being a total time derivative, can have no effect on the classical dynamics of the system. Yet, it has profound effects on tunneling, giving rise to the Aharonov-Casher phase and the resulting interference effect.

To see this, let us consider the two tunneling paths illustrated schematically by the arrows in Fig. 2. The end points of the two paths are equivalent, since they merely differ in θ by 2π . Every term in Eq. (4) except the last is symmetric under the reflection $\theta \rightarrow -\theta$. Hence, the actions of the two paths differ only because of the last term. The imaginary-time action for each path can then be separated into two parts: $S_I^j = \tilde{S}_I + S_{\text{geo}}^j$, where \tilde{S}_I is the action obtained by using all but the last term in Eq. (4) and is the same for both paths. The geometric-phase action for each path is given by

$$S_{\text{geo}}^{1,2} = -iq \left(\frac{\Phi_0}{2\pi} \right) \int_{1,2} \dot{\theta} d\tau = \mp iq \left(\frac{\Phi_0}{2\pi} \right) \pi = \mp i\pi(q/2e)\hbar. \quad (5)$$

From Eq. (2), the tunnel splitting is given by

$$\Delta = 2\Delta_0 \cos(q\pi/2e), \quad (6)$$

where $\Delta_0 = \omega_0 e^{-\tilde{S}_I/\hbar}$ is the tunnel splitting associated with one path. The explicit forms of parameters ω_0 and \tilde{S}_I are not relevant for the present calculation; they will be presented in subsequent work.

The upshot of Eq. (6) is that the tunnel splitting for flux tunneling can be modulated by the application of a gate voltage. When the induced charge is e (half a Cooper pair), the two tunneling paths interfere completely destructively and flux tunneling is wholly suppressed. Furthermore, when $e < q \bmod(4e) < 3e$ the tunnel splitting becomes negative. This means that the ground and excited states interchange roles: if the ground (excited) state can be approximated by $(|\phi_0\rangle + (-)|\phi_1\rangle)/\sqrt{2}$ when $-e < q \bmod(4e) < e$, where $|\phi_0\rangle$ and $|\phi_1\rangle$ are the distinct fluxoid states connected by the tunneling paths in Fig. 2, then it becomes $(|\phi_0\rangle - (+)|\phi_1\rangle)/\sqrt{2}$ for $e < |q| \bmod(4e) < 3e$.

In the limit of large ‘‘internal’’ charging energy, $E_C \gg E_J$, the physics of suppression of flux tunneling for $q \approx e$ remains the same as for $E_C \ll E_J$. The quantitative form of the tunneling amplitude is, however, quite different because for $E_C \gg E_J$ the system wave function is delocalized in the θ direction and flux trajectories with all values of $\Delta\theta$ contribute to the tunneling rate. Since the regime of flux tunneling requires the ‘‘external’’ charging energy $E_Q = (2e)^2/2C$ to be smaller than E_J , all energies of the flux dynamics are then smaller than the energies of the charge dynamics on the central island. In this case, when $q \approx e$, only the two charge states of the island, $n = 0$ and $n = 1$, are relevant for the charge dynamics. The Hamiltonian of the system reduces to

$$H = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_x)^2}{2L} - E_+ \cos(\pi\Phi/\Phi_0)\sigma_z + \frac{e(q - e)}{C_\Sigma} \sigma_x - E_- \sin(\pi\Phi/\Phi_0)\sigma_y, \quad (7)$$

where $E_\pm \equiv (E_{J1} \pm E_{J2})/2$ and the Pauli matrices σ_i act on the basis of $(|0\rangle \pm |1\rangle)/\sqrt{2}$, the symmetric and antisymmetric superpositions of the charge states.

The potential $U(\Phi)$ for the evolution of the flux Φ is then seen to have two branches, depending on the charge-space state of the system: For $\Phi < \Phi_0/2$, the symmetric charge state $(|0\rangle + |1\rangle)/\sqrt{2}$ provides the lower branch of the potential, whereas for $\Phi > \Phi_0/2$, the sign of $\cos(\pi\Phi/\Phi_0)$ changes and the lower branch of the potential is given by the antisymmetric state $(|0\rangle - |1\rangle)/\sqrt{2}$. This means that to avoid suppression of flux tunneling from a state localized in the region $\Phi < \Phi_0/2$ into the state with $\Phi > \Phi_0/2$, the system should make a transition between the symmetric and antisymmetric charge states in the course of flux tunneling. From the wave function of these states in the θ representation: $|\psi_+\rangle \propto \cos(\theta/2)$ and $|\psi_-\rangle \propto \sin(\theta/2)$, one can see that such a transition

corresponds to the shift in θ by $\pm\pi$, analogous to the similar shift in the limit of large E_J .

The Hamiltonian (7) shows that the coupling between the two branches of the potential is provided by the difference E_- of the Josephson energies and deviations of the induced charge q from e . In the absence of coupling, transitions between the two potential branches and, as a result, flux tunneling, are suppressed. When the coupling is weak (i.e., E_- and $q - e$ are small), transitions between the potential branches take place near the degeneracy point $\Phi = \Phi_0/2$ and can be described in terms of Landau-Zener tunneling. The only difference from the standard situation of Landau-Zener tunneling is that the transition is not driven by a classical external force but by the flux motion under the tunnel barrier and therefore can be described as occurring in imaginary time. The amplitude of such an imaginary-time Landau-Zener transition has been found in the context of the dynamics of Andreev levels in superconducting quantum point contacts [25]. Adapted for the present problem, the expression for the transition amplitude is

$$w = \frac{1}{\Gamma(\lambda)} \left(\frac{2\pi}{\lambda}\right)^{1/2} \left(\frac{\lambda}{e}\right)^\lambda e^{i\phi}, \quad (8)$$

$$\lambda \equiv \frac{[e(q - e)/C_\Sigma]^2 + E_-^2}{2E_+[E_Q E]^{1/2}}.$$

Here Γ denotes the Gamma function and E is the absolute value of the energy of the quasistationary flux state at $\Phi < \Phi_0/2$ measured relative to the effective top of the potential barrier formed by the crossing at $\Phi = \Phi_0/2$ of the two branches of the potential. The phase ϕ of the tunneling amplitude coincides with the phase of the coupling between the potential branches in the Hamiltonian (7) and is given by

$$\tan\phi = \frac{E_- C_\Sigma}{e(q - e)}. \quad (9)$$

Since the overall amplitude of flux tunneling is proportional to w and since $w(\lambda) \approx (2\pi\lambda)^{1/2}$ for $\lambda \ll 1$, Eq. (8) shows that, when E_- is small, flux tunneling is suppressed at $q \approx e$. Equation (9) also shows that, similarly to the limit of large E_J , the sign of the tunneling amplitude at $E_- \rightarrow 0$ changes abruptly when q moves through the point $q = e$. The absolute value of amplitude w (8) is shown in Fig. 3 as a function of charge q for several values of E_- . The curves in Fig. 3 indicate that the suppression of the flux tunneling amplitude at $q \approx e$ remains pronounced for quite a large degree of asymmetry of the Josephson coupling energies.

The suppression of tunneling occurs for arbitrary E_J/E_C , as can be seen from the following argument. When $E_{J1} = E_{J2}$, $\Phi_x = \Phi_0/2$, and $q = e$, the Hamiltonian (1) has two symmetries, which lead to the degeneracy of the energy eigenstates and hence the suppression of tunneling. The Josephson coupling terms in the Hamiltonian can be written as $-2E_J \cos(\pi\Phi/\Phi_0) \cos\theta = -E_J \cos(\pi\Phi/\Phi_0) \sum_n (|n\rangle\langle n+1| + |n+1\rangle\langle n|)$. Then

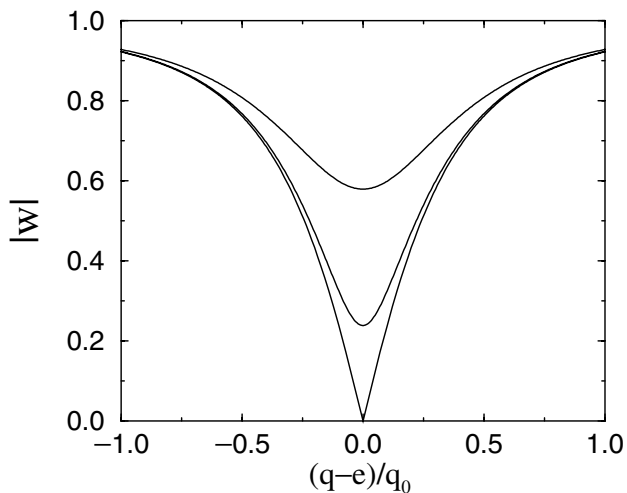


FIG. 3. Absolute value of the transition amplitude between the two branches of the flux potential, which is proportional to the amplitude of flux tunneling, as a function of the induced charge q at $q \approx e$. Note that the scale $q_0 \equiv C_{\Sigma}(2E_+)^{1/2}(E_Q E)^{1/4}/e$ of variations of q is much smaller than e . From bottom to top, the curves correspond to increasing difference between the Josephson energies of the two junctions: $E_-/(2E_+)^{1/2}(E_Q E)^{1/4} = 0.0, 0.1, \text{ and } 0.3$.

the Hamiltonian is symmetric with respect to the following two transformations: (I) $|n\rangle \rightarrow (-1)^n |n\rangle$, $\Phi \rightarrow \Phi_0 - \Phi$, which requires that $\Phi_x = \Phi_0/2$, and (II) $n \rightarrow 1 - n$, which requires that $q = e$. We show that one cannot construct an energy eigenfunction that is simultaneously an eigenfunction of both of these transformations and, therefore, that each energy eigenfunction must be degenerate with another one with which it is related by the application of both transformations. Let us write the energy eigenfunction in Φ, n space: $|\Psi(n, \Phi)\rangle = \sum_n \psi_n(\Phi) |n\rangle$. Transformation (I) gives $\psi_n(\Phi) = (-1)^n \psi_n(\Phi_0 - \Phi)$, i.e., $\psi_n(\Phi)$ is an odd (even) function of $\Phi - \Phi_0/2$ if n is odd (even). Transformation (II) requires that $\psi_n(\Phi) = \psi_{1-n}(\Phi)$, but this is impossible because it equates an even function with an odd one. Thus, at $\Phi_x = \Phi_0/2$ and $q = e$, the eigenfunctions of Hamiltonian (1) are all twofold degenerate for an arbitrary E_J/E_C ratio.

Our results are a variant of the Aharonov-Casher effect applied to superconductors, such as that studied by Reznik and Aharonov [5]. We note that in our calculation, the relevant charge is not n , which may not even be a good quantum number, but q , the charge induced by the gate voltage. In addition, in our scenario the amount of flux that tunnels need not be a multiple of Φ_0 , as noted above and illustrated in Fig. 2. Unlike the case studied in [5] where fluxons of flux Φ_0 move around a localized, external charge, the modulation of flux tunneling in the SQUID

does not require that Φ tunnel between quantized values or that the charge on the island be localized.

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