Using Algebraic Geometry, *first edition*

November 23, 2005

Comment on Maple code:

The Maple code in the first edition of *Using Algebraic Geometry* uses commands from the `grobner` package. Examples include the `gbasis` and `finite` commands.

Soon after the publication of the first edition, the `grobner` package was replaced with an updated version, the `Groebner` package. In the new version, some commands (such as `gbasis`) have a different syntax, while other (such as `finite`) have different names. This means that the Maple code in the first edition will need to be modified if the reader wants to run it under more recent versions of Maple.

In the second edition of *Using Algebraic Geometry*, we updated the Maple code in the text so that it is consistent with the `Groebner` package.

Errata:

Page vii, line -7: Replace "to point out the reader" with "to point the reader"

Page viii, line 10: Replace "explanantions" with "explanations"

Page viii, line -10: Replace "more efficient that" with "more efficient than"

Page x, line 14: Add the following new sentence at the end of the paragraph:

Special thanks go to Rainer Steinwandt for his careful reading of the first printing.

Page xi, line -7: Replace "Equations Via Resultants" with "Equations via Resultants"

Page 4, line 3: Insert a "." at the end of the display

Page 7, line 22: Replace "e.g." with "e.g.,"

Page 8, line 16: Replace "leftmost" with "left-most"

Page 9, line below (2.5): replace "where $a_i, r \in k[x_1, \dots, x_n]$" with "where $a_i, r \in k[x_1, \dots, x_n]$, for each i, $a_i f_i = 0$ or $\textsc{lt}_>(f) \geq \textsc{lt}_>(a_i f_i)$"

Page 11, line 5: Replace the period with a question mark

Page 12, line 11: Replace "Chapter 1 §4" with "Chapter 1, §4"

Page 13, line -5: Replace "$\{g_1, \dots, g_t\} \subset I$" with "$\{g_1, \dots, g_t\}$"

Page 13, line -4: Replace "of I" with "of $I = \langle g_1, \dots, g_t \rangle$

Page 15, line -9: Replace "In fact" with "In fact,"

Page 16, line 18: Replace "Given $f(t)$ an" with "Given $f(t)$, an"

Page 23, line 3: Replace "**Exercise 11.**" with "**Exercise 11.** Let k be algebraically closed."

Page 23, lines 3 and 4 of part (a) of Exercise 11: Delete "let k be an infinite field,"

Page 23, line 1 of part (c) of Exercise 11: Replace "show that any algebraic variety" with "show that $\mathbf{V}(I) \setminus \mathbf{V}(J) \subset \mathbf{V}(I:J)$ and that any variety"

Page 27, line 10: Replace "`gbasis(PList,VList,plex);`" with "`gbasis(PList2,VList2,plex);`"

Page 27, line -1: Replace "in in" with "in"

Page 28, line -16: Replace "For example" with "For example,"

Page 29, line 1: Replace "Exercise 11" with "Exercise 10"

Page 31, line 10: Replace "one variable" with "one-variable"

Page 32, line -12: Replace "Exercise 9" with "Exercise 8"

Page 32, line -10: Replace "Exercise 10" with "Exercise 9"

Page 33, line -3: Replace "root multiplicity" with "root of multiplicity"

Page 36, line 9: Replace "section will" with "section we will"

Page 36, line -5: Replace "be be" with "be"

Page 37, line -27: Replace "finite dimensional" with "finite-dimensional"

Page 38, line -12: Replace "in which $c_j \in k$" with "in which the $c_j \in k$"

Page 51, line 16: Replace "in particular" with "in particular,"

Page 55, line 2: Replace "that that" with "that"

Page 59, lines 6 and 8: Replace "$m_{\mu j}$" with "m_{mj}" (twice)

Page 59, line 20: Replace "Proposition (2.8)" with "Proposition (2.7)"

Page 61, lines 3 and 4: Replace "for each j the jth column of Q is a left eigenvector of M and the jth diagonal entry" with "each nonzero column of Q is a left eigenvector of M and the corresponding diagonal entry"

Page 61, lines 6 and 7: Replace "for each i the ith column of P is a right eigenvector of M and the ith diagonal entry of" with "each nonzero row of P is a left eigenvector of M and the corresponding diagonal entry of"

Page 61, line -7: Replace "Proposition (2.8)" with "Proposition (2.7)"

Page 65, lines -6 to -4: Delete these lines and replace them with the following:

as defined in Chapter 4. Hence we will only sketch the proof in the special case when I is radical. By Theorem (2.10), this means that $\mathbf{V}(I) = \{p_1, \ldots, p_m\}$, where m is the dimension of the algebra A. Given the basis $B = \{[x^{\alpha(i)}]\}$ of A, Proposition (4.7) implies that $(p_j^{\alpha(i)})$ is an invertible matrix.

Page 67, line -1: Replace "$8xz$." with "$8xz$"

Page 69, line 9: Replace "in the case" with "in this case"

Page 69, line -1: Replace "$-\mathrm{rem}(p_{i-1}(t), p_{i-2}(t), t)$" with "$-\mathrm{rem}(p_{i-2}(t), p_{i-1}(t), t)$"

Page 70, line 1: Replace "division of p_{i-1} by p_{i-2}" with "division of p_{i-2} by p_{i-1}"

Page 72, line 4: Replace "$x^3 + 4x - 1$" with "$x^3 + x - 1$"

Page 72, line 4: In rows 3 and 4 of the determinant in display (1.3), replace "4" with "1" (twice)

Page 72, second bullet: Replace "have a common" with "have a nontrivial common"

Page 74, line -8: Replace "Exercise 7 of Chapter 2, §2" with "Exercise 18 of Chapter 2, §4"

Page 74, line -7: Replace "the smaller" with "smaller"

Page 74, line -3: Replace "*homogenous*" with "*homogeneous*"

Page 77, part b of Exercise 11: In the Hint, replace "Proposition (2.4)" with "Proposition (4.2)"

Page 78, line 5: Replace "use basis" with "use the basis"

Page 78, line 14: Replace "well defined" with "well-defined"

Page 79, line -3: Replace "coefficents" with "coefficients"

Page 80, line 10: Replace "$\mathbf{C}[x_1, \ldots, x_n]$" with "$\mathbf{C}[x_0, \ldots, x_n]$"

Page 80, line -17: Replace "$F_i = \sum_{i=0}^n c_{ij} x_j$" with "$F_i = \sum_{j=0}^n c_{ij} x_j$"

Page 80, line -16: Replace "discussion" with "the discussion"

Page 81, line -7: Replace "parametrizaton" with "parametrization"

Page 82, line 6: Replace "point that" with "point out that"

Page 84, line -15: Replace "nontrival" with "nontrivial"

Page 84, line -1: Replace "chosing" with "choosing"

Page 85, line -9: Replace "homogenegous" with "homogeneous"

Page 87, line 6: Replace "satisfing" with "satisfying"

Page 87, line 2 of Exercise 8: Replace "nontrival" with "nontrivial"

Page 87, line -3: Replace "that coefficient" with "that the coefficient"

Page 88, line -19: Replace "is a" with "is an"

Page 89, line 18: Replace "that final" with "that the final"

Page 90, line -19: Replace "finite dimensional" with "finite-dimensional"

Page 91, line 7 and 8: Replace "degree" with "degrees"

Page 91, line -10: The exponent of λ should be "$d_0 \cdots d_{j-1} d_{j+1} \cdots d_{n-1}$"

Page 92, line -14: Replace "$[i_1 \ldots i_n]$" with "$[i_0 \ldots i_n]$"

Page 92, line -7: Replace "homogenous" with "homogeneous"

Page 93, line 10: Replace "$F_3 =$" with "$F_0 =$"

Page 95, line 4: Replace "chose" with "choose"

Page 95, line -13: Insert a "." at the end of the display

Page 97, line 14: Replace "n sets" with "$n+1$ sets"

Page 99, line 5: Replace "repacing" with "replacing"

Page 99, line 5: Replace "multplying" with "multiplying"

Page 99, line 9: Replace "the coefficents" with "the coefficients"

Page 101, line -6: Replace "columns the" with "columns of the"

Page 103, line 13: Replace "auxilliary" with "auxiliary"

Page 103, line 20: Replace "univeral" with "universal"

Page 104, line 23: Replace "a $N \times N$" with "an $N \times N$"

Page 106, line -10: Replace " ,and that assume" with ", and assume"

Page 106, lines -10 and -9: Replace "homogenous" with "homogeneous"

Page 107, line -13: Replace "nontrival" with "nontrivial"

Page 107, line -11: Replace "nontrival" with "nontrivial"

Page 108, line 11: Replace "§5 Solving Equations Via Resultants" with "§5 Solving Equations via Resultants"

Page 108, lines -16 and -15: Replace "multipicities" with "multiplicities"

Pages 109, 111, and 113, running head: Replace "§5. Solving Equations Via Resultants" with "§5. Solving Equations via Resultants"

Page 114, line -13: Replace "says" with "say"

Pages 115 and 117, running head: Replace "§5. Solving Equations Via Resultants" with "§5. Solving Equations via Resultants"

Page 116, line -1: Replace "$A = \mathbf{C}(u)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_n\rangle$" with "$A = \mathbf{C}(u)[x_1, \ldots, x_n]/\langle u - x_n, f_1, \ldots, f_{n-1}\rangle$"

Page 123, line 19: Replace "$\widehat{A} = \mathbf{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_n\rangle$" with "$\widehat{A} = \mathbf{C}(u_0)[x_1, \ldots, x_{n-1}]/\langle \hat{f}_1, \ldots, \hat{f}_{n-1}\rangle$"

Page 118, line -3: Replace "signficance" with "significance"

Page 119, running head: Replace "§5. Solving Equations Via Resultants" with "§5. Solving Equations via Resultants"

Page 119, line 8: Replace "have solutions of multiplicity" with "have multiplicity"

Page 119, line -10: Replace "part c" with "parts a and b"

Page 120, line -1: Replace "hidden variable" with "the hidden variable"

Page 121, running head: Replace "§6. Solving Equations via Eigenvalues" with "§5. Solving Equations via Resultants"

Page 121, line 6: Replace "in proof" with "in the proof"

Page 121, line 8: Replace "homogenous" with "homogeneous"

Page 123, line 3: Replace "than" with "that"

Page 123, line 4: Replace "n sets" with "$n + 1$ sets"

Page 123, display nearest to top of page: Replace "$|\alpha| = d$" with "$|\gamma| = d$" (three times)

Page 124, line 5: Replace "," after displayed formula with "."

Page 124, line 11: In (6.3), insert a "." at end of display

Page 125, line 10: Replace "$\mu = d_1, \ldots, d_n$" with "$\mu = d_1 \cdots d_n$"

Page 127, line 2: Replace "eigenvalues methods" with "eigenvalue methods"

Page 129, line -10: Replace "substitiuting" with "substituting"

Page 135, line -6: Replace "$v(x+y) \leq \min\{v(x), v(y)\}$" with "$v(x+y) \geq \min\{v(x), v(y)\}$"

Page 138, line 1: Replace "equal" with "are equal"

Page 139, line -18: At the end of the line, insert the new sentence "Theorem (2.2) below will guarantee that $m(p)$ is finite."

Page 139, line -17: Replace "solution" with "a solution"

Page 139, line -1: Replace "part d" with "part c"

Page 140, line -15: Replace "the the" with "the"

Page 141, line 6: Replace "a isomorphism" with "an isomorphism"

Page 142, line 16: Replace "note that that" with "note that the"

Page 142, line -7: Replace "using part b of the lemma" with "since part b of the lemma implies $e_i \in I\mathcal{O}_j$ for $i \neq j$ and $e_i - 1 \in I\mathcal{O}_i$"

Page 144, line -10: Replace "Theorem (5.4) of Chapter 3" with "Proposition (5.8) of Chapter 3"

Page 145, lines -15 and -14: Replace "$u^6 - 18x^4 + 81x^2 - 108$" with "$u^6 - 18u^4 + 81u^2 - 108$"

Page 147, line 14: Replace "will say singular" with "say that the singular"

Page 147. line -13: Replace "$\mathbf{C}\{x_1, \ldots, x_n\}\langle \partial f/\partial x_1, \ldots, \partial f/\partial x_n\rangle$" with "$\mathbf{C}\{x_1, \ldots, x_n\}/\langle \partial f/\partial x_1, \ldots, \partial f/\partial x_n\rangle$"

Page 149, line 16: Replace "note that that" with "note that the"

Page 149, line -14: Replace "*eigenspace* of of" with "*eigenspace* of"

Page 150, line 9: Replace "Proposition (5.9) of Chapter 2" with "Exercise 12 and Proposition (5.8) of Chapter 3"

Page 150, line -14: Replace "$\det(\partial^2 f/\partial x_i \partial x_j) \neq 0$" with "$\det(\partial^2 f/\partial x_i \partial x_j)\big|_{(x_1,\ldots,x_n)=(0,\ldots,0)} \neq 0$"

Page 150, line -5: Replace "one" with "One"

Page 152, line 2: Replace "satifies" with "satisfies"

Page 154, line 17: Replace "$S = \{1 + g \ : \ \mathrm{LT}(g) < 1\}$" with "$S = \{1 + g \ : \ g = 0 \text{ or } \mathrm{LT}(g) < 1\}$"

Page 154, line -1: Insert a space on either side of the equals sign, so that "$>_{mixed}\!=\!>_U$" becomes "$>_{mixed} = >_U$"

Page 156, line 9: Replace "in is" with "is"

Page 157, line -4: Replace "for f:" with "for f:"

Page 157, line -3: Replace "$(-x) \cdot f_5$" with "f_5"

Page 157, lines -2 and -1: Delete and replace with the following:

However, we also have

$$f_5 = 3x^6 = 3x^5 \cdot x = 3x^5 \cdot \frac{x + x^2}{1 + x} = \frac{3x^5}{1+x} f.$$

Page 158, lines 1–7: Delete and replace with the following:

Backsubstituting this into the previous equation for f and multiplying by $1+x$, we obtain

$$(1+x)f = (1+x)g + (1+x)(x - 2x^2 + 2x^3 - 3x^4)f + 3x^5 f,$$

Then moving xf to the right-hand side gives an equation of the form

Page 159, lines 5 and 7: Replace "$t^a > t^{a'}x^\beta$" with "$t^a >' t^{a'}x^\beta$" (twice)

Page 159, line -8: Replace both "–" with "—"

Page 160, line 14: Replace the entire display with "$a + \deg(F) = \deg(A_i) + \deg(F_i) = \deg(H)$"

Page 160, line -17: Replace "$k[x_1, \ldots, x_n]$" with "$k[t, x_1, \ldots, x_n]$"

Page 160, line -4: Replace "$M := \{G \in L : \text{LT}(G) | \text{LT}(t^a H)$ for some $a\}$" with "$H := F$; $L := \{F_1, \ldots, F_s\}$; $M := \{G \in L : \text{LT}(G) | \text{LT}(t^a H)$ for some $a\}$"

Page 160, line -2: Replace "$G \in L$" with "$G \in M$"

Page 161, line 25: Replace "U ,$A_{i,j}$" with "U, $A_{i,j}$"

Page 161, line 30: Replace "$G = H_j$ for some $j < k$" with"$G = H_k$ for some $k < j$"

Page 161, display above line -6: In the first line of the display, replace "$j \neq i$" with "$\ell \neq i$", and in the second line of the display, replace "$j = i$" with "$\ell = i$"

Page 162, line 5: Replace "Since $t^{a_j - a_k}\text{LT}(H_k) > \text{LT}(H_j)$, the leading term of U_{j+1} is" with "Since $t^{a_j - a_k}\text{LT}(H_k) > \text{LT}(H_j)$ and $M \cdot \text{LT}(H_k) = t^a \text{LT}(H_j)$, one obtains $t^{a+a_j} > Mt^{a_k}$, so that $\text{LT}(U_{j+1}) =$"

Page 162, line 17: Replace ", but (not" with "(but not"

Page 162, line -10: Replace "If we write" with "Write"

Page 162, lines -9 and -8: Replace these lines with the following:

u' is a unit in R. Then homogenize $f', f_1, \ldots, f_s$ using a new variable t. This gives homogeneous polynomials in $k[t, x_1, \ldots, x_n]$. If we apply Theorem (3.10) and then dehomogenize, we obtain

Page 164, part b of Exercise 12: Add a period after the displayed matrix

Page 164, lines -18 to -12: Delete all of Exercise 13.

Page 164, line -10: Insert "Hint: See the discussion following (3.7)."

Page 164, line -7: Replace "on one" with "any one"

Page 164, line -5: Replace "the *the*" with "the"

Page 164, line -1: Replace "$\text{LT}(f_1), \ldots \text{LT}(f_s)$" with "$\text{LT}(f_1), \ldots, \text{LT}(f_s)$"

Page 165, line 6: Replace "$\text{LT}(g_1), \ldots \text{LT}(g_t)$" with "$\text{LT}(g_1), \ldots, \text{LT}(g_t)$"

Page 165, lines 14 and 18: Replace "Exercise 4" with "Exercise 3" (twice)

Page 166, lines 5 and 7: Replace "$t^a > t^{a'}x^\beta$" with "$t^a >' t^{a'}x^\beta$" (twice)

Page 166, line 11: Replace "G" with "S"

Page 166, line -17: Replace "Exercise 4" with "Exercise 3"

Page 166, line -12: Replace "Buchberger's" with "of Buchberger's"

Page 166, line -8: Replace "computing" with "computed"

Page 167, line -19: Replace "Exercise 7" with "Exercise 6"

Page 167, lines -12 and -11: Replace "`http://www.mathematik.uni-kl.de/z̃ca/Singular/`" with "`http://www.singular.uni-kl.de/`"

Page 169, line 10: Replace "also long as" with "as long as"

Page 169, line 15: Replace "Exercise 8" with "Exercise 7"

Page 169, line 19: Replace "Exercise 4" with "Exercise 3"

Page 169, line -1: Replace "Exercise 9" with "Exercise 8"

Page 170, line 10: Replace "don't depend on" with "doesn't depend on"

Page 170, line 15: Replace "$V \in k^n$" with "$V \subset k^n$"

Page 170, line -17: Replace "Exercise 13" with "Exercise 12"

Page 171, line 6: Replace "$h_4 = d(v-u) - (c-u)w$," with "$h_4 = d(v-u) - (c-u)w = 0$,"

Page 171, line 9: Replace "$-2cd$" with "$-2bd$" in the formula for g_1

Page 173, line 6: Replace "imples" with "implies"

Page 173, line -12: Replace "work the" with "work in the"

Page 174, line 14: Replace "$I \cap k[x_1, \ldots, x_n]$" with "$I \cap k[x_1, \ldots, x_n]_{\langle x_1, \ldots, x_n \rangle}$"

Page 174, line -15: Replace "Exercise 4" with "Exercise 3"

Page 174, line -11: Replace "and and" with "and"

Page 175, line 14: Replace "The one can" with "Then one can"

Page 175, line -16: Replace "$f, g \in J$" with "$f, g \in R$"

Page 175, line -4: Replace "**Exercise 3**" with "**Exercise 2**"

Page 176: Renumber **Exercises 4–9** as **Exercises 3–8**

Page 176, line 11: Replace "if and only" with "if and only if"

Page 176, line 16: Replace "that set" with "that the set"

Page 176, line 19: Replace "for local local orders, using Exercise 5" with "for local orders, using Exercise 4"

Page 177: Renumber **Exercises 10–13** as **Exercises 9–12**

Page 177, line 6: Replace "$\langle f_1, \ldots f_n \rangle$" with "$\langle f_1, \ldots, f_n \rangle$"

Page 177, line 8: Replace "$f_0, \ldots, f_n$" with "$f_1, \ldots, f_n$"

Page 177, line 10: Replace "$f_1 = \ldots = f_n = 0$" with "$f_1 = \cdots = f_n = 0$"

Page 177, line -15: Replace "The curves" with "The curve"

Page 178, line 3: Replace "**Exercise 14**" with "**Exercise 13**"

Page 178, line 3: Replace "a r-dimensional" with "an r-dimensional"

Page 178, line -13: Replace "$f =$" at end of line with "$f_t =$"

Page 180, line -24: Replace "are can be written" with "can be written"

Page 181, line 7: Replace "diferent" with "different"

Page 181, line -24: Replace "$\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$" with "$\{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$"

Page 181, line -23: Replace "exist nonzero $a_1, a_2, a_3 \in R = k[x, y, z]$ such that" with "exist $a_1, a_2, a_3 \in R = k[x, y, z]$, not all zero, such that"

Page 181, line -11: Replace "additive identity" with "the additive identity"

Page 182, line 4: Replace "indicate column" with "indicate the column"

Page 182, line 8: Replace "usefl" with "useful"

Page 183, line -16: Replace "need show" with "need to show"

Page 183, line -11: Replace "define maps" with "defines maps"

Page 184, line 9: Replace "$a_n\mathbf{f}_n$" with "$a_l\mathbf{f}_l$"

Page 184, line -19: Replace "exerise 1" with "Exercise 1"

Page 184, line -18: Replace "homomorhism" with "homomorphism"

Page 184, line -17: Replace "matrix with" with "matrix A with"

Page 187, line 7: Replace "exercise" with "Exercise"

Page 187, line 8: Replace "ker$A$" with "ker A"

Page 187, line -9: Besides the 1994 paper [PW] by Park and Woodburn, we should also mention two other papers that deal with algorithmic aspects of the Quillen-Suslin result:

L. Caniglia, G Cortiñas, S. Danón, J. Heintz, T. Krick and P. Solernó (working group N. Fitchas), *Algorithmic aspects of Suslin's proof of Serre's gonjecture*, Comput. Complexity **3** (1993), 31–55

N. Fitchas and A. Galligo, *Nullstellensatz effectif et conjecture de Serre (théorème de Quillen-Suslin) pour le calcul formel*, Math. Nachr. **149** (1990), 231–253.

Page 187, line -4: Replace "α_m" with "a_m"

Page 188, line -26: Replace "no proper subset generates" with "no proper subset generates ker A"

Page 190, line 1: Replace "of of" with "of"

Page 190, line -17: Replace "me_l" with "$\mathbf{e}_l$"

Page 190, line -12: Replace "span module" with "span the module"

Page 190, line -4: Replace "Show that" with "Show that the"

Page 192, line 8: Replace "exercise 2" with "Exercise 2"

Page 192, line -15: Replace "coeficients" with "coefficients"

Page 192, line -19: Replace "Exercise 13" with "Exercise 15"

Page 193, line 19: Replace "Exercise 21a" with "Exercise 21b"

Page 193, line 29: Replace "in e) above" with "in part a"

Page 193, line −18: Replace "We let $R = k[x, y]$" with "Let $R = k[x, y]$, where k is a field of characteristic different from 2,"

Page 203, lines −11 to −3: Delete and replace with the following:

a. Verify that $\mathbf{f} = (f_1, f_2, f_3)^T = (1, -x/2, -1/2)^T \in R^3$ satisfies $(1+x)f_1 + (1-y)f_2 + (x+xy)f_3 = 1$.
b. Let I be the 3×3 identity matrix. Verify that the columns $\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$ of the matrix $I - \mathbf{f} \cdot A$ span $\ker A$. Hint: If $A\tilde{\mathbf{f}} = 0$, then $\tilde{\mathbf{f}} = (I - \mathbf{f} \cdot A)\tilde{\mathbf{f}}$ is a linear combination of the columns of $I - \mathbf{f} \cdot A$.
c. Show that $\{\mathbf{g}_1, \mathbf{g}_2\}$ is a basis of $\ker A$. (Unfortunately, the result of part c is special to the choice of $\mathbf{f}$ made in part a. If $\mathbf{f}$ is an arbitrary solution of $A\mathbf{f} = 1$, then the first two columns of $I - \mathbf{f} \cdot A$ need not give a basis of the kernel.)

Page 194, line 1: Replace "and an" with "an"

Page 195, line −13: Replace "and the its" with "and that its"

Page 195, line −11: Replace "entries zero), Let" with "entries zero). Let"

Page 196, lines −19 and −16: Replace "Exercise 20" with "Exercise 29" (twice)

Page 196, line −3: Replace "$f_1{}'$" with "f_1"

Page 196, line −2: Replace "submodule of" with "submodule"

Page 197, line 12: Replace "coefficents" with "coefficients"

Page 197, line −15: Replace "of A" with "if A"

Page 199, line 24: Replace "Exercise 2 of §1" with "Exercise 4 of §1"

Page 200, lines −16 and −15: Replace "(see Exercise 5 below)" with "(see Exercise 12 of §3)"

Page 200, line −2: Replace "a ordering" with "an ordering"

Page 200, line −1: Replace "is *monomial ordering*" with "is a *monomial ordering*"

Page 205, line 4: Replace "$k = [x, y]$" with "$R = k[x, y]$"

Page 205, line 7: Insert a "." at the end of the second line of the displayed formula

Page 207, lines 10 and 11: Replace "$\sum_{j=1}^{n}$" with "$\sum_{j=1}^{m}$" (twice)

Page 209, lines −14, −12 and −7: Replace "$> \ldots >$" with "$> \cdots >$" (three times)

Page 209, line −5: Replace "Exercise 9b" with "Exercise 8b"

Page 212, line −7: Replace "with respect any" with "with respect to any"

Page 212, line −3: Replace "Exercise 1" with "Exercise 2"

Page 213, lines 13 to 22: Replace these lines with the following:

$m_v \epsilon_v$ for some v. With this v fixed, we set

$$\mathbf{s} = \sum_{u \in S} m_u \epsilon_u$$

where $S = \{u : m_u \mathrm{LT}_>(\mathbf{g}_u) = m_v \mathrm{LT}_>(\mathbf{g}_v)\}$.

One can show without difficulty that $\mathbf{s}$ is an element of $\mathrm{Syz}(\{\mathrm{LT}_>(\mathbf{g}_u) : u \in S\})$. By part (c) of Proposition (2.3) of this chapter, it follows that $\mathbf{s}$ is an element of the submodule of R^s generated by the

$$\sigma_{uw} = \frac{\mathbf{m}_{uw}}{\mathrm{LT}_>(\mathbf{g}_u)}\epsilon_u - \frac{\mathbf{m}_{uw}}{\mathrm{LT}_>(\mathbf{g}_w)}\epsilon_w$$

where $u < w$ are elements of S. Then (3.4) implies that $\mathrm{LT}_{\mathcal{G}}(\mathbf{s})$ is divisible by $\mathrm{LT}_{\mathcal{G}}(\mathbf{s}_{ij})$ for some $i < j$. So by definition the $\mathbf{s}_{ij}$ form a Gröbner basis for M with respect to the $>_{\mathcal{G}}$ order. $\square$

Page 213, line -4: Replace "$n \times t$ and $n \times s$ matrices" with "$m \times t$ and $m \times s$ matrices"

Page 214, line 8: Replace "$n = 1$" with "$m = 1$"

Page 214, line -3: In the 2×2 matrix BA, replace the bottom row "0 1" with "$x/2$ 1"

Page 215, line 7: Replace "mf_t" with "$\mathbf{f}_t$"

Page 216, line 11: Replace "the a" with "a"

Page 217, line 7: Replace "Exercise 11" with "Exercise 13"

Page 217, line 8: Replace "not alone sufficient" with "alone not sufficient"

Page 217, line 14: Replace "it seems" with "seems" and "the the" with "the"

Page 217, line -9: Replace "$d_s\mathbf{f}_s$" with "$d_s\mathbf{g}_s$"

Page 217, line -8: Replace "submodule of" with "submodule"

Page 218, line 2: Replace "$d_s\mathbf{f}_s$" with "$d_s\mathbf{g}_s$"

Page 219, line 18: Replace "R^{n+t+s}" with "R^{m+t+s}"

Page 219, line -14: Replace "Proposition (3.8)" with "Proposition (3.11)"

Page 221, running head: Replace "§4. Modules over Local Rings" with "§3. Computing Syzygies"

Page 221, line -3: Replace "a ordered" with "an ordered"

Page 223, line -10: Replace "well" be "well-"

Page 224, line 11: Replace "for for" with "for"

Page 224, line 21: Replace "over a" with "over"

Page 225, line 15: Replace "$\{\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_2$ that" with "$\{\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3\}$ that"

Page 225, line -14: Replace "$a_1f_1 + \ldots + a_sf_s$" with "$a_1f_1 + \cdots + a_sf_s$"

Page 225, line -13: Replace "$a_1 \ldots, a_s$" with "$a_1, \ldots, a_s$"

Page 225, line -12: Replace "$a_1f_1 + \ldots + a_{s-1}f_{s-1}$" with "$a_1f_1 + \cdots + a_{s-1}f_{s-1}$"

Page 226, line -6: Replace the period at the end of the sentence with a question mark.

Page 226, line -5: Replace "need little lemma" with "need a little lemma"

Page 227, line -4: Replace "have have" with "have"

Page 231, line 6: Replace "such that such" with "such"

Page 231, lines 16 and 17: Replace "some s" with "for some s"

Page 233, line 11: Replace "$F_1(M) \subset \ldots F_{s+1}(M)$" with "$F_1(M) \subset \cdots \subset F_{s+1}(M)$"

Page 236, line 4: Replace "are also conveniently" with "is also conveniently"

Page 236, line 5: Replace "homormorphisms" with "homomorphisms"

Page 238, line -9: Replace "$y\mathbf{s}_{12} - \mathbf{s}_{13} + x\mathbf{s}_{23}$" with "$(y-1)\mathbf{s}_{12} - \mathbf{s}_{13} + x\mathbf{s}_{23}$"

Page 239, line 11: Replace "is a ℓ" with "is an ℓ"

Page 239, line -1: Replace "syzygy" with "the syzygy"

Page 242, line 1: Replace "the the" with "the"

Page 242, line -8: Replace "ϕ" with "φ"

Page 245, line -7: Replace "a R-module" with "an R-module"

Page 247, line 10: Replace "Exercise 7" with "Exercise 8"

Page 247, line -14: Replace "Exercise 8" with "Exercise 9"

Page 248, line -6: Replace "Exercise 10" with "Exercise 11"

Page 249, line -10: Replace "from (1.8)" with "from (1.7)"

Page 249, line -5: Replace "see (1.16)" with "(see (1.14))"

Page 255, line 4 of Exercise 2: Replace "$\deg f_{i1} + d_1 = \cdots = \deg f_{im} + d_m$" with "$\deg f_{i1} - d_1 = \cdots = \deg f_{im} - d_m$"

Page 258, line -10: Replace "minimial" with "minimal"

Page 261, line 12: Replace "$\varphi_\ell(v_1)$" with "$\varphi_\ell(e_1)$"

Page 261, line -8: Replace "a R-linear" with "an R-linear"

Page 262, line 4: Replace "$+a_m c_m$" with "$+a_m d_m$"

Page 262, line 8: Replace "$a_1c_1 + a_2d_2 + \cdots + a_mc_m$" with "$a_1e_1 + \cdots + a_me_m$"

Page 262, line -18: Replace "that that" with "that"

Page 263, line -4: Replace "homogenous" with "homogeneous"

Page 265, line 2: Replace "$v - g(v)$" with "$v - \psi(v)$"

Page 265, line 13: Replace "same notation of that proof" with "same notation as in that proof"

Page 266, line -20: Replace "of first" with "of the first"

Page 266, line -2: Replace "If M finitely" with "If M is a finitely"

Page 267, line -15: Replace "$H_R(s)$" with "$H_R(t)$"

Page 268, line 15: Replace "is [CLO]" with "in [CLO]"

Page 270, line -14: Replace "$(D/d!)t^m$" with "$(D/d!)t^d$"

Page 270, line -4: Replace "We've see" with "We've seen"

Page 271, line -6: Replace "that one might" with "than one might"

Page 272, line 12: In formula (4.10), replace "$\to 0.$" with "$\to 0,$"

Page 274, line 1: Replace "see show" with "show"

Page 274, line -16: Replace "intersection the moving" with "intersection of the moving"

Page 275, line -7: Replace "im(β)" with "im(α)"

Page 277, line 3: In formula (4.19), replace "$\to 0$." with "$\to 0$"

Page 278, line 22: Replace "a $(n-\mu)\times(n-\mu)$" with "an $(n-\mu)\times(n-\mu)$"

Page 279, line 2: Replace "knowning" with "knowing"

Page 279, line 3: Replace "of quotient" with "of the quotient"

Page 279, line -6: Replace "Proposition (4.24)" with "Proposition (4.22)"

Page 280, line 7: Replace "a m-dimensional" with "an m-dimensional"

Page 282, line 13: Replace "$g \cdot f(x,y) = f(-y,x)$" with "$g \cdot f(x_1,x_2) = f(-x_2,x_1)$"

Page 282, line -19: Replace "Note that y_2" with "Note that y_1"

Page 283, line -12: Replace "where how the" with "how the"

Page 283, line -2: Replace "homogenous" with "homogeneous"

Page 284, line -18: Replace "algebaically" with "algebraically"

Page 284, line -23: Replace "sygygies" with "syzygies"

Page 285, line 2: Replace "area" with "areas"

Page 285, line -13: Replace "$a(t),/c(t)$" with "$a(t)/c(t)$"

Page 286, line 3: Relace "$\to 0$." with "$\to 0$"

Page 289, line 11: Replace "$+\cdots d_s a_s$" with "$+\cdots+d_s a_s$"

Page 294, line -1: Replace "conection" with "connection"

Page 296, line -3: Replace "Similary" with "Similarly"

Page 297, line 6: Replace "$\lambda_i m_i$" with "$\lambda_i s_i$"

Page 298, line 8: Replace "Exercise 11" with "Exercise 12"

Page 299: line -18: Insert a "." after the matrix displayed in equation (2.4)

Page 299, line -13: Replace "multiplying the each" with "multiplying each"

Page 300, line 13: Replace "occuring" with "occurring"

Page 300, line 20: Replace "$a_n t^{m_l}$" with "$a_l t^{m_l}$"

Page 304, line -10: Replace "right corresponds to [013][023], and the one on the left" with "left corresponds to [013][023], and the one on the right"

Page 307, line -17: In the display defining $X_{\mathcal{A}}$, replace "$\overline{\phi\big((\mathbf{C}^*)^n\big)}$" with "$\overline{\phi_{\mathcal{A}}\big((\mathbf{C}^*)^n\big)}$"

Page 309, line 8: In the last line of formula (3.6), replace "$= 0$." with "$= 0$,"

Page 311, line -10: Replace "uv, us, vt, st" with "uv, ut, sv, st"

Page 311, line -8: Replace "shows" with "show"

Page 312, line 1: In the statement of Theorem (3.13), replace "$\mathcal{A} = \{m_1,\ldots,m_l\} \subset \mathbf{Z}^n_{\geq 0}$" with "$\mathcal{A} = \{m_1,\ldots,m_l\} \subset \mathbf{Z}^n$"

Page 314, line 17: Insert a "." after the displayed matrix

Page 316, lines 1 and 2: Replace "for all vertices" with "for at least one vertex"

Page 318, line 6: Replace "$b_{30}y^3$" with "$b_{03}y^3$"

Page 318, line -12: Replace "$P_1, \ldots, P_r$ in $\mathbf{R}^n$" with "$P_1, \ldots, P_r \subset \mathbf{R}^n$"

Page 319, line 1: Replace "a n-dimensional" with "an n-dimensional"

Page 320, line 1: Replace "and line" with "and the line"

Page 320, line -12: Replace "used the equation" with "used in the equation"

Page 321, line 5: Replace "poltyopes" with "polytopes"

Page 322, line 18: Replace "homogenous" with "homogeneous"

Page 325, line -19: Replace "$\{w_1, ..., w_n\}$" with "$\{w_1, ..., w_{n-1}\}$"

Page 326, line -16: Insert a "." at end of displayed formula

Page 326, lines -15 and -14 (part (c) of Exercise 16): Delete these lines.

Page 326, line -5: Insert a "." at end of displayed formula

Page 328, line 8: Replace "we that" with "that we"

Page 328, line -7: Replace "in section" with "in the section"

Page 329, line 3: Replace "where learned" with "where we learned"

Page 329, line -3: Replace "$\mathcal{A}_n$" with "$\mathcal{A}_l$"

Page 330, line -20: Replace "there a way" with "there is a way"

Page 331, line -7: Replace "as distinct" with "has distinct"

Page 334, line -1: Replace "$(P_2)_\mu$" with "$(P_2)_\nu$"

Page 339, lines 14–15: Replace "is take a known solutions" with "is to take a known solution"

Page 339, line -16: Replace "continuous a" with "a continuous"

Page 343, line -20: Replace "formulas for" with "formulas for the"

Page 344, line 6 of Definition (6.5): Add the following sentence: "Furthermore, if $R_j = F_1' + \cdots + F_m'$ is another cell in the subdivision, then $R_i \cap R_j = (F_1 \cap F_1') + \cdots + (F_m \cap F_m')$."

Page 345, lines -5 and -4: Replace "subdivsion" with "subdivision"

Page 346, line -14: Replace "**Exericse 4.** Work out the inequalties" with "**Exercise 4.** Work out the inequalities"

Page 347, line -1: Replace "the" with "with the"

Page 348, line -11: Replace "Chapter 3.6.7" with "Chapter 3."

Page 352, line 9: Replace "implies is" with "implies"

Page 353, line -15: Replace "$u_0 \ldots, u_n$" with "$u_0, \ldots, u_n$"

Page 355, line -3: Replace "x^{β_n}" with "x^{β_μ}"

Page 359, line -14: Replace "section 2.8" with "Section 2.8"

Page 359, line -4: Replace "and and" with "and"

Page 361, line 2: Replace "written the same" with "written in the same"

Page 364, line -2: Replace "$z_1^{b_1} \cdots z_n^{b_n}$" with "$z_1^{b_1} \cdots z_m^{b_m}$"

Page 371, line 9: Replace "ismorphism" with "isomorphism"

Page 371, line 15: Replace "$a'_{ij} = e_j - a_{ij}$" with "$a'_{ij} = e_j + a_{ij}$"

Page 371, line 16: Replace "for all i" with "for all i"

Page 371, line -6: Replace "$t^e z_1^{b'_1} \cdots z_n^{b'_n}$" with "$t^e z_1^{b'_1} \cdots z_m^{b'_m}$"

Page 372, line -19: Replace "giving" with "give"

Page 374, line -5: Replace "to the find" with "to find"

Page 381, line -5: Replace "the the" with "the"

Page 391, line -3: Replace "both both" with "both"

Page 393, line 10: Replace "term the right" with "term on the right"

Page 395, line -6: Replace "Theorem 4.4" with "Lemma 3.3 and Theorem 3.5"

Page 396, line -2: Remove the "." after the displayed matrix

Page 397, line 6: Replace "$g_3 = (2xy^2$," with "$g_3 = (2xy^2 + y^3$,"

Page 397, line 7: Replace with the following two lines:

$$\begin{aligned} g_4 &= (-3xy^2 - 2y^3, x^3 - 3xy^2 - 2y^3, 0, 0, x, -x + 2y, 0, 3x + 2y) \\ g_5 &= (x^2y^2, 0, 0, 0, -y^2, 0, 0, -x^2). \end{aligned}$$

Page 397, line -8: Replace "$\sigma_1 =$" with "σ_1"

Page 398: line 2: In the first display at the top of the page, insert a "." after the displayed matrix

Page 398, line 9: In the second display at the top of the page, insert a "." at the end of the fourth equation

Page 399, line -11: Add the following sentence: "The connection between splines and syzygies was first noted by Schumacher in 1979."

Page 400, line 11: Replace "f_i well-defined" with "f_i is a well-defined"

Page 401, line -5: In the displayed formula, replace "$6\binom{k-1}{2}$." with "$6\binom{k-1}{2}$"

Page 404, line 9: Insert a "." at end of displayed formula

Page 406, line -12: Replace "$v_1, \ldots v_q$" with "$v_1, \ldots, v_q$"

Page 406, line -4: Replace "the the" with "the"

Page 409, line -20: Replace "this–every" with "this—every"

Page 416, line 27: Replace "a extension" with "an extension"

Page 421, line 16: Replace "the these" with "the"

Page 432, line 5: At the very beginning of the line, replace "y" with "αy"

Page 432, line 6: Replace "the same" with "is the same"

Page 434, line 19: Replace "$a^{\ell+\delta-2}$" with "$\alpha^{\ell+\delta-2}$"

Page 434, line 23: Replace "$a^{2(\ell+j)},\ldots,a^{(q-2)(\ell+j)}$" with "$\alpha^{2(\ell+j)},\ldots,\alpha^{(q-2)(\ell+j)}$"

Page 434, line 24: Replace "matrix H matrix" with "matrix H"

Page 434, line -3: Replace "$a^{\ell+\delta-2}$" with "$\alpha^{\ell+\delta-2}$"

Page 442, line -10: Replace "are are" with "which are"

Page 446, line 12: Replace "inverses give" with "inverses gives"

Page 447, line 11: Replace "the the" with "the"

Page 448, line 17: Replace "process." with "process,"

Page 450, line -4: Replace the eight displayed points with the following:

$$\begin{array}{rclcrcl} P_1 &=& (0,0) & \qquad & P_2 &=& (0,1) \\ P_3 &=& (1,\alpha) & & P_4 &=& (1,\alpha^2) \\ P_5 &=& (\alpha,\alpha) & & P_6 &=& (\alpha,\alpha^2) \\ P_7 &=& (\alpha^2,\alpha) & & P_8 &=& (\alpha^2,\alpha^2). \end{array}$$

Page 452, line 3: Replace "$(0,1,0)$" with "$(1,0,0)$"

Page 452, line -1: Replace "and equation" with "an equation"

Page 453, line 3: Replace "$|a_i|$" with "$|\alpha_i|$"

Page 454, line 1: Replace "above makes" with "makes"

Page 454, line -8: Replace "that the the" with "that the"

Page 455, line 6: Replace "show" with "Show"

Page 455, line 14: Replace "Hint: what" with "Hint: What"

Page 455, line -15: Replace "dependences" with "dependencies"

Page 455, line -4: Replace "ideal" with "the ideal"

Page 457, line -6: Delete "However, this equals the"

Page 457, lines -5 to -1: Delete these lines completely

Page 458, lines -18 and -17: Replace "determine" with "determining"

Page 462, line 8: Replace "$\alpha,\alpha^2\ldots,\alpha^{i_0-1}$" with "$\alpha,\alpha^2,\ldots,\alpha^{i_0-1}$"

Page 463, line -19: Replace "have codeword" with "have a codeword"

Page 463, line -13: Replace "for the number for the number" with "for the number"

Page 463, line -1: Insert a "." at end of displayed equation

Page 464, line -6: Replace "be" with "been"

Page 467, line -17: Replace "${}^{a}H_I$" with "${}^{a}HF_I$"

Page 469, line 15: In entry [BoF], replace "Konvexen" with "konvexen"

Page 469, line 16: In entry [BoF], replace "Chelsea, New York, 1971" with "Chelsea, New York, 1971 and Springer-Verlag, New York, 1974"

Page 471, line 11: Replace "Base" with "bases"

Page 471, lines 12 and 13: Replace "`http://www.mathematik.uni-kl.de/~zca/Singular/`" with "`http://www.mathematik.uni-kl.de/~zca/`"

Page 472, line −14: Replace "A. Pfister" with "G. Pfister"

Page 473, line 23: In entry [Roj4], replace "*refinement of of*" with "*refinement of*"

Page 473, line −9: In entry [Schre1], replace "Weierstrass'chen" with "Weierstraßschen"

Page 478, column 2, line 2: Replace "$[i_1, \ldots, i_n]$" with "$[i_0, \ldots, i_n]$"