Ideals, Varieties and Algorithms, *second edition*

Errata for the ***second, third, fourth*** *and* ***sixth*** *printings as of June 11, 2007*

Note: The fifth printing of the second edition of *Ideals, Varieties and Algorithms* is defective. If you have a copy of this printing, please send email to David Cox at `dac@cs.amherst.edu`.

Page viii, line 5: Replace "The follows chart" with "The following chart"

Page 1, line -1: Replace "$x_1^{\alpha_1} \cdot x_2^{\alpha_2} \ldots x_n^{\alpha_n}$" with "$x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdots x_n^{\alpha_n}$"

Page 4, line 16: Replace "two polynomials are equal" with "two polynomials over an infinite field are equal"

Page 4, line -17: Replace "Proposition 4" with "Proposition 5"

Page 5, line 12: Replace "a nonzero polynomial $\mathbb{F}_p[x]$" with "a nonzero polynomial in $\mathbb{F}_p[x]$"

Page 31, line 6: Replace "Exercise 5" with "Exercise 6"

Page 31, line -4: Replace "*for all*" with *for all*"

Page 35, line 9: Replace "$k[x, y]$" with "$\mathbb{Q}[x, y]$"

Page 45, line 24 (part a of Exercise 11): Replace "Theorem 6" with "Theorem 7"

Page 51, part (a) of Exercise 5, first line: Replace "$x^e y^f$" with "$x^a y^b$"

Page 51, part (b) of Exercise 6, display: Replace "$[f(t)]^e [g(t)]^f$" with "$[f(t)]^a [g(t)]^b$"

Page 51, line -4: Replace "$e + f \leq m$ form a linearly *dependent* set" with "$a + b \leq m$ are linearly *dependent*"

Page 55, line 21: Replace "$\alpha(l) > \alpha(l+1)$" with "$\alpha(l) >_{lex} \alpha(l+1)$" .

Page 56, lines 10 and 11: Replace with the following:

$$|\alpha| = \textstyle\sum_{i=1}^n \alpha_i > |\beta| = \sum_{i=1}^n \beta_i, \quad \textit{or} \quad |\alpha| = |\beta| \textit{ and the rightmost nonzero entry of } \alpha - \beta \in \mathbb{Z}^n \textit{ is negative.}$$

Page 56, line 15: Replace "$\alpha - \beta$" with "$(1, 5, 2) - (4, 1, 3)$"

Page 58, line -7: Replace "It it" with "Is it"

Page 66, lines 1 and 2: Replace Exercise 4 with the following new exercise:

4. Let $f = a_1 f_1 + \cdots + a_s f_s + r$ be the output of the division algorithm.

   a. Complete the proof begun in the text that multideg$(f) \geq$ multideg$(a_i f_i)$ when $a_i f_i \neq 0$.

   b. Prove that multideg$(f) \geq$ multideg(r) when $r \neq 0$.

Page 66, between lines 2 and 3: Insert a blank line.

Page 72, part b of Exercise 12: Replace "the gradlex order $>_{gradlex}$" with "the grlex order $>_{grlex}$"

Page 72, line -7: Replace "$x_i, \ldots, x_i$" with "$x_1, \ldots, x_i$"

Page 73, line -3: Replace "$g_1, \ldots, g_s \in I$" with "$g_1, \ldots, g_t \in I$"

Page 74, line 5: Replace "$g_1, \ldots, g_s \in I$" with "$g_1, \ldots, g_t \in I$"

Page 74, line 13: Replace "g_s" with "g_t" (twice)

Page 74, lines 15–19: Replace "g_s" with "g_t" (five times)

Page 74, line 17: Replace "$\langle \mathrm{LT}(g_1), \ldots \mathrm{LT}(g_t)\rangle$" with "$\langle \mathrm{LT}(g_1), \ldots, \mathrm{LT}(g_t)\rangle$"

Page 74, line 19: Replace "$\langle g_1, \ldots, g_t\rangle$" with "$(g_1, \ldots, g_t)$"

Page 74, lines -9, -8, -7 and -3: Replace "g_s" with "g_t" (four times)

Page 75, lines 1, 8 and 11: Replace "g_s" with "g_t" (three times)

Page 78, lines 18, 20, 21 and 23: Replace "g_s" with "g_t" (4 times)

Page 79, line 24: Replace "Show that $\mathbb{R}^3$" with "Show that in $\mathbb{R}^3$"

Page 82, line 4: Replace "$(c_1d_1 + \cdots + c_{t-1}d_{t-1})(p_{t-1} - p_t) + (c_1d_1 + \cdots + c_td_t)p_t$" with "$(c_1d_1 + \cdots + c_{s-1}d_{s-1})(p_{s-1} - p_s) + (c_1d_1 + \cdots + c_sd_s)p_s$"

Page 82, line 10: Replace "$(c_1d_1 + \cdots + c_{t-1}d_{t-1})S(f_{t-1}, f_t)$" with "$(c_1d_1 + \cdots + c_{s-1}d_{s-1})S(f_{s-1}, f_s)$"

Page 92, line 15: Replace "basis" with "basis of I".

Page 92, line -18: Replace "and $n \times m$" with "an $n \times m$"

Page 93, line 10: Replace "g_s" with "g_t"

Page 93, line 16: Replace "$= x^2y^2$" with "$= -x^2y^2$"

Page 93, line -14: Replace this display with the following:

$$f = (-4xy^2z - 4y^4) \cdot f_1 + 0 \cdot f_2 + 0 \cdot f_3 + 0 \cdot f_4 + (-3) \cdot f_5 + 0.$$

Page 94, line -3: Replace "$-\frac{36717}{590}z^4$" with "$+\frac{36717}{590}z^4$"

Page 96, line -12: Replace "it make sense" with "it makes sense"

Page 99, line -11: Replace "$a + b + c = 3$" with "$a + b + c = 3$,"

Page 99, line -10: Replace "$a^2 + b^2 + c^2 = 5$" with "$a^2 + b^2 + c^2 = 5$,"

Page 100, line 1 of **Definition 1**: Replace "$\{g_1, \ldots, g_s\}$" with "$\{g_1, \ldots, g_t\}$"

Page 100, line 1 of **Lemma 2**: Replace "$\{g_1, \ldots, g_s\}$" with "$\{g_1, \ldots, g_t\}$"

Page 101, line 1 of **Theorem 3**: Replace "$\{g_1, \ldots, g_s\}$" with "$\{g_1, \ldots, g_t\}$"

Page 104, lines 12 and 13: Replace "Continuing in this way, we can write" with "Since a nonzero syzygy must have at least two nonzero components, continuing in this way will eventually enable us to write"

Page 104, line -6: Replace "g_s" with "g_t"

Page 105, line 1: Replace "g_s" with "g_t"

Page 106, line 1: Replace "g_s" with "g_t"

Page 106, line -14: Replace "$B := \{(i, j)|$" with "$B := \{(i, j) :$"

Page 106, line -2: Replace "$B := B \cup \{(i, t)|$" with "$B := B \cup \{(i, t) :$"

Page 108, line 19: Replace "*section*" with "*selection*"

Page 109, line -3: Replace "$\mathrm{LT}(G_s)$" with "$\mathrm{LT}(g_s)$"

Page 110, line 3: Replace "$n \times (n+1)$" with "$m \times (m+1)$"

Page 110, line 4: Replace "$n \times n$" with "$m \times m$"

Page 110, line 10: Replace "$(n+1) \times (n+1)$" with "$(m+1) \times (m+1)$"

Page 117, line -1: Replace "§4" with "§5"

Page 118, line 7 of Exercise 2: Replace "smallest field k" with "smallest field k containing $\mathbb{Q}$"

Page 118, line -8: Replace "$k[x_l, \ldots, x_n]$" with "$k[x_1, \ldots, x_n]$"

Page 119, line 3: Replace "$<_l$" with "$>_l$"

Page 119, line 21: Replace "$>_1$" with "$>_1$ (this is $>_l$ with $l = 1$)"

Page 122, line -2: Replace "$N_i > 0$" with "$N_i \geq 0$"

Page 127, line -6: In the formula for g_4, replace "$-2y^2$" with "$+2y^2$"

Page 129, line -1: In the display, the letters x, y, t should be in math italic, and the same is true for the letters m and n which appear in subscripts. Thus the display should read:

$$\pi_{m+1}(y, t_1, \ldots, t_m, x_1, \ldots, x_n) = (x_1, \ldots, x_n),$$

Page 133, lines -4 and -3: Delete the sentence "Since we over $\mathbb{R}$ and the denominators never vanish, you can use Exercise 13."

Page 143, line 6 of Exercise 2: Replace "nonzero real number λ" with "nonzero number $\lambda \in k$"

Page 143, third line of Exercise 4: Replace "$-\frac{\partial}{\partial y}$" with "$+\frac{\partial}{\partial y}$"

Page 143, line 1 of Exercise 5: Add the new sentence "Assume that $\mathbb{Q} \subset k$."

Page 147, line -1: Replace "A or B" with "$\tilde{A}$ or $\tilde{B}$"

Page 152, line 3: In the displayed equation, insert "det" between the first equal sign and the large left parenthesis

Page 153, line -11: In this display, replace "polynomialin" with "polynomial in"

Page 155, line 12: Replace "Proposition 1" with "Proposition 2"

Page 155, second display of Exercise 6: Replace "$f = f_1^{r_1} \cdots f_l^{r_l}$" with "$f = c f_1^{r_1} \cdots f_l^{r_l}, \quad c \in k$,"

Page 155, hint of Exercise 8: Replace "Use the previous exercise." with "Use Exercise 6 (you may assume $\mathbb{Q} \subset k$)."

Page 156, first sentence of part c of Exercise 14: The period should be a question mark.

Page 163, line 16: Replace "F divides $u_2 f_2(x_1, \mathbf{c})$" with "F divides $u_2 f_2(x_1, \mathbf{c}) + \cdots + u_s f_s(x_1, \mathbf{c})$"

Page 165: The running head at the top of the page should be "§6. Resultants and the Extension Theorem"

Page 168, line 22: Replace "$g \cdot 1 = g \in I$" with "$g = g \cdot 1 \in I$"

Page 169, line -15: Replace "degreee" with "degree"

Page 169, line -12: Replace "subspace" with "subspace of k^n"

Page 170, line 16: Replace "that constant, $g_2, \ldots, g_t$ can be" with "that constant, so that $g_2, \ldots, g_t$ can be"

Page 170, line -13: Delete the word "affine"

Page 171, line 4: Replace "Given a polynomial" with "Given a nonzero polynomial"

Page 171, line 15: Replace "$f(a_1, \ldots a_n) = 0$" with"$f(a_1, \ldots, a_n) = 0$"

Page 172, line 18: Replace "a polynomial f" with "a nonconstant polynomial f"

Page 173, lines 5–7: Replace "A k-linear map which satisfies this property and the additional property that it maps the constant polynomial 1 to the constant polynomial 1" with "As we will see in Definition 8 of Chapter 5, §2, a map between rings which preserves addition and multiplication and also preserves the multiplicative identity"

Page 173, line 15: Replace "$k[x_1, \ldots, x_n]$" with "$k[\tilde{x}_1, \ldots, \tilde{x}_n]$"

Page 173, line -6: Replace "any" with "some"

Page 175, line -11: Replace "function from varieties" with "function which takes varieties"

Page 176, line -13: Replace "$I \subset \tilde{I}$" with "$1 \in \tilde{I}$"

Page 177, display on line -11: Replace "$f = f_1^{a_1} \cdots f_r^{a_r}$" with "$f = cf_1^{a_1} \cdots f_r^{a_r}, \quad c \in k$,"

Page 177, line -3: Replace "$f = f_1^{a_1} \cdots f_r^{a_r}$" with "$f = cf_1^{a_1} \cdots f_r^{a_r}$"

Page 178, line 5: Replace "This," with "Thus,"

Page 178, line -6: Replace "unique to muliplication" with "unique up to muliplication"

Page 180, line 14: Replace "folllowing" with "following"

Page 182, illustration at top of page: The x and y axes are labelled incorrectly. First, the old "y" should be changed to "x". Second, the old "x" should be deleted and, at the opposite end of that axis, a "y" should be added.

Page 182, line -7: Replace "$\mathbf{V}(f_1, \ldots f_r)$" with "$\mathbf{V}(f_1, \ldots, f_r)$"

Page 183, line -11: Replace "If I and J be ideals" with "If I and J are ideals"

Page 184, line 18: Replace "$k[x, y]$" with "$\mathbb{Q}[x, y]$"

Page 186, lines 12, 17 and 23: Replace "$k[x, y]$" with "$\mathbb{Q}[x, y]$" (three times)

Page 187, line 10: Replace "implies that the following" with "implies the following"

Page 188, line -14: Replace "$f^m f^p = f^{m+p}$" with "$f^{m+p} = f^m f^p$"

Page 188, line -11: Replace "Show that" with "Show that in $\mathbb{Q}[x, y]$, we have"

Page 189, line 5: Replace "$I^k \subset J$ for some integer $k > 0$" with "$I^\ell \subset J$ for some integer $\ell > 0$"

Page 189, line -14: Replace "$I_1, \cdots I_r$" with "$I_1 \cdots I_r$"

Page 189, line -12: Replace "Let I be an ideal" with "Let I, J be ideals"

Page 189, lines -7 and -6: Replace "$\alpha_A f$" with "$\alpha_A(f)$" (three times)

Page 189, line -1: Replace "$k[x_1, \ldots, x_m]$" with "$k[y_1, \ldots, y_n]$"

Page 190, between lines -3 and -2: Insert the following new paragraph:

We also note that $I(\overline{S}) = I(S)$. The inclusion $I(\overline{S}) \subset I(S)$ follows from $S \subset \overline{S}$. Going the other way, $f \in I(S)$ implies $S \subset \mathbf{V}(f)$. Then $S \subset \overline{S} \subset \mathbf{V}(f)$ by Definition 2, so that $f \in I(\overline{S})$.

Page 191, line 12: Replace "$x_1, \ldots x_l$" with "$x_1, \ldots, x_l$"

Page 191, line 16: Replace "$W = V(K)$" with "$W = \mathbf{V}(K)$"

Page 194, line -16: Replace "is a radical" with "is radical"

Page 194, lines -8 and -7: Replace "$\alpha_A f$" with "$\alpha_A(f)$" (three times)

Page 194, line -5: Replace "$\alpha_a^{-1}(I')$" with "$\alpha_A^{-1}(I')$"

Page 195, line 12: Replace "Proposition 8" with "Proposition 8 of §2"

Page 195, line -14: Replace "$V(xz, yz)$" with "$\mathbf{V}(xz, yz)$"

Page 197, line -3: Replace "Problem 11" with "Exercise 11".

Page 197, line -1: Insert spaces before and after "if and only if"

Page 198, line -3: Replace "$1/b \cdot b = 1$" with "$1 = 1/b \cdot b$"

Page 199, lines -15 to -11: Replace with the following:

$(a_1, \ldots, a_n) \in \mathbf{V}(I)$. This means that every $f \in I$ vanishes at $(a_1, \ldots, a_n)$, so that $f \in \mathbf{I}(\{(a_1, \ldots, a_n)\})$. Thus, we can write

Page 199, line -5: Replace "relies heavily on the Nullstellensatz" with "uses the Weak Nullstellensatz"

Page 199, line -4: Replace "the Nullestellensatz" with "the Weak Nullstellensatz"

Page 200, line 8: Replace "such that if $J\,K \subset I$," with "such that $J\,K \subset I$,"

Page 201, lines -15, -14 and -12: Replace "union" with "finite union" (three times)

Page 203, line 13: Replace "Exercise 8" with "Theorem 7 of §4"

Page 203, line -11: Replace "The V" with "Then V"

Page 204, line -9: Replace "minimal representation" with "minimal decomposition"

Page 204, line -8: Replace "$\{I : f : f \in k[x_1, \ldots, x_n]\}$" with "$\{I : f \ : \ f \in k[x_1, \ldots, x_n]\}$"

Page 206, line 21: Replace "$J = \langle xz - y^2, x^3 - yz, z^2 - x^2 y\rangle$" with "$J = \langle xz - y^2, x^3 - yz, z^2 - x^2 y\rangle \subset k[x, y, z]$, where k is infinite"

Page 206, line -18: Replace "$+A(x)+$" with"$+xA(x)+$"

Page 206, line -15: Replace "$I = \langle xz - y^2, z^3 - x^5\rangle$" with "$I = \langle xz - y^2, z^3 - x^5\rangle \subset \mathbb{Q}[x, y, z]$"

Page 206, line -8: Replace "$f_s^{a_s}$" with "$f_r^{a_r}$"

Page 207, line -14: Replace "is in" with "is an"

Page 208, line -1: Replace "$\{\sqrt{I : f} : f \in k[x_1, \ldots, x_n]\}$" with "$\{\sqrt{I : f} \ : \ f \in k[x_1, \ldots, x_n]\}$"

Page 209, line 10: Replace "$\{I : f : f \in k[x_1, \ldots, x_n]\}$" with "$\{I : f \ : \ f \in k[x_1, \ldots, x_n]\}$"

Page 209, line -14: Replace "$\langle x, y^2\rangle \in$" with "$\langle x, y^2\rangle \subset$"

Page 209, line -8: Replace "$I : g^n \subset I : g^{n+1}$ for all $n \geq 1$" with "$I : g^m \subset I : g^{m+1}$ for all $m \geq 1$"

Page 211, line 6: Replace "$\mathbf{I}\cap$" with "$I\cap$"

Page 214, line 11: Replace "$1 \leq i \leq m$" with "$1 \leq i \leq n$"

Page 217, line -19: Replace "affine variety" with "variety"

Page 219, line -15: Replace "$f - g + g - h = f - h \in I$" with "$f - h = f - g + g - h \in I$"

Page 221, line -6: Replace "$\Phi([f)]$" with "$\Phi([f])$"

Page 222, line -20: Replace "$\mathbf{V}(I)$" with "$\mathbf{I}(V)$"

Page 223, line 3 of the statement of Proposition 10: Replace "$I \subset J \subset R$" with "$I \subset J \subset k[x_1, \ldots, x_n]$"

Page 223, line -6: Replace "$0 \in I \in J$" with "$0 \in I \subset J$"

Page 225, line -14: Replace "for any ring homomorphism," with "for any ring homomorphism $\phi : R \to S$,"

Page 225, line -5: Replace "every class" with "every class in"

Page 227, line -12: The "R" in "$\mathbb{R}[x, y]$" should be in the blackboard bold font

Page 229, line 3: Replace "$\in k[x, y, z]$" with "$\subset k[x, y, z]$"

Page 230, line -16: Replace "$\mathbb{C}$" with "$\mathbb{C}^n$"

Page 230, line -8: Replace "$\textsc{lm}(g)$ for some $g \in G$" with "$\textsc{lm}(g_i)$ for some $g_i \in G$"

Page 231, line 5: Replace "some $g \in G$, such that $\textsc{lt}(g)$" with "some $g_i \in G$, such that $\textsc{lt}(g_i)$"

Page 231, line 6: Replace "$\textsc{lt}(g)$" with "$\textsc{lt}(g_i)$"

Page 232, line 2: Replace "$I = \{y - x^7, x^{12} - x\}$" with "$I = \langle y - x^7, x^{12} - x\rangle$"

Page 234, lines -14 and -12: Replace "(3)" with "(4)" (twice)

Page 235, line 13: Replace "$\langle \textsc{lt}(I)\rangle$" with "$\langle \textsc{lt}(J)\rangle$"

Page 236, line -10: Replace "Then $\mathbf{V}(J) \subset V$" with "Then $\mathbf{V}(\widetilde{J}) \subset V$"

Page 237, line -18: Replace "$[f] \in \mathbf{I}_V(\mathbf{V}_V(\widetilde{J}))$" with "$[f] \in \mathbf{I}_V(\mathbf{V}_V(J))$"

Page 237, line -9: Replace "algebraic varieties" with "affine varieties"

Page 237, line -7: Replace "$\alpha \circ \beta = \mathrm{id}_W$, and $\beta \circ \alpha = \mathrm{id}_V$" with "$\alpha \circ \beta = \mathrm{id}_W$ and $\beta \circ \alpha = \mathrm{id}_V$"

Page 238, line -14: Replace "the surface $Q = \mathbf{V}(F_{-1})$" with "the surface $Q = F_{-1}$"

Page 238, line -13: Replace "as is *any* graph of a polynomial function" with "as is the graph of *any* polynomial function"

Page 240, line 10: Replace "V" with "V"

Page 240, line -3: Replace "$[F] = [f(y_1, \ldots, y_n)]$" with "$[F] = [F(y_1, \ldots, y_n)]$"

Page 241, line 21; Replace "$(\phi) \circ \mathrm{id}_W$" with "$\phi \circ \mathrm{id}_W$"

Page 242, line 8: Replace "consider linear" with "consider the linear"

Page 242, line -7: Replace "corresponding to π" with "corresponding to α"

Page 243, line 5: Replace "$\alpha^*(y^5 - x^2)$" with "$\alpha^*([y^5 - x^2])$"

Page 244, line 17: Replace "Q_1-" with "$Q_1 =$"

Page 244, line 1 of Exercise 10: Replace "isomorphism" with "homomorphism"

Page 245, line 4: Replace "$y = \beta(g(t))$" with "$y = \alpha(g(t))$"

Page 249, line 2: Replace "is not defined is $\mathbf{V}_V(g_1, \ldots, g_n)$, a proper" with "is defined includes $V - \mathbf{V}_V(g_1 \cdots g_n) = V - (\mathbf{V}_V(g_1) \cup \cdots \cup \mathbf{V}_V(g_n))$, where $\mathbf{V}_V(g_1 \cdots g_n)$ is a proper"

Page 249, line 10: Replace "$f_t k_t - h_t g_t$" with "$f_i k_i - h_i g_i$"

Page 249, line 16: Replace "$\mathbf{V}_V(g_1, \ldots, g_n)$" with "$\mathbf{V}_V(g_1 \cdots g_n)$"

Page 249, line 17: Replace "$\mathbf{V}_V(k_1, \ldots, k_n)$" with "$\mathbf{V}_V(k_1 \cdots k_n)$"

Page 249, lines -9 and -8: Replace "$\psi \circ \phi$" with "$\phi \circ \psi$" (twice)

Page 249, line -8: Replace "identity map 1_W" with "identity map id_W"

Page 250, display on line -19: Replace the entire display with "$V' = \mathbf{V}_V([Q_1 \cdots Q_l g_1 \cdots g_l]) \subset V$"

Page 250, line -6: Replace "W and W" with "W and $\psi : W$"

Page 250, line -5: Replace "1_W" with "id_W"

Page 251, line 5: Replace "Theorem 8" with "Theorem 9"

Page 251, line 11: Replace "1_W" with "id_W"

Page 251, line 17: Replace "$\psi(W - W')$" with "$\psi(W - W_1)$"

Page 251, line -19: Replace "Definition 5" with "Definition 4"

Page 251, line -11: Replace "$1_{k(V)}$" with "$\mathrm{id}_{k(V)}$"

Page 251, line -9: Replace "identity of the" with "identity on the"

Page 251, line -5: Replace "components" with "complements"

Page 252, line -20: Replace "cublic" with "cubic"

Page 252, line -19: Replace "§1" with "§3"

Page 252, line -6: Replace "$(1, 0, 0)$" with "$(0, 0, 1)$"

Page 252, lines -5 and -4: Replace "define a rational mapping $\phi : S - - \to \mathbb{R}^2$ by setting $\phi(q) = L_q \cap W$" with "show that $\phi(q) = L_q \cap W$ defines a rational mapping $\phi : S - - \to \mathbb{R}^2$"

Page 253, Exercise 8: Replace this exercise with the following:

8. In Exercise 10 of §1, you showed that there were no nonconstant polynomial mappings from $\mathbb{R}$ to $V = \mathbf{V}(y^2 - x^3 + x)$. In this problem, you will show that there are no nonconstant *rational* mappings either, so V is not birationally equivalent to $\mathbb{R}$. In the process, we will need to consider polynomials with complex coefficients, so the proof will actually show that $\mathbf{V}(y^2 - x^3 + x) \subset \mathbb{C}^2$ is not birationally equivalent to $\mathbb{C}$ either. The proof will be by contradiction.
   a. Start by assuming that $\alpha : \mathbb{R} - - \to V$ is a nonconstant rational mapping defined by $\alpha(t) = (a(t)/b(t), c(t)/d(t))$ with a and b relatively prime, c and d relatively prime, and b, d monic. By substituting into the equation of V, show that $b^3 = d^2$ and $c^2 = a^3 - ab^2$.
   b. Deduce that $a, b, a + b$, and $a - b$ are all squares of polynomials in $\mathbb{C}[t]$. In other words, show that $a = A^2$, $b = B^2$, $a + b = C^2$ and $a - b = D^2$ for some $A, B, C, D \in \mathbb{C}[t]$.
   c. Show that the polynomials $A, B \in \mathbb{C}[t]$ from part b are nonconstant and relatively prime and that $A^4 - B^4$ is the square of a polynomial in $\mathbb{C}[t]$.

d. The key step of the proof is to show that such polynomials cannot exist using *infinite descent*. Suppose that $A, B \in \mathbb{C}[t]$ satisfy the conclusions of part c. Prove that there are polynomials $A_1, B_1, C_1 \in \mathbb{C}[t]$ such that

$$\begin{aligned} A - B &= A_1^2 \\ A + B &= B_1^2 \\ A^2 + B^2 &= C_1^2. \end{aligned}$$

e. Prove that the polynomials A_1, B_1 from part d are relatively prime and nonconstant and that their degrees satisfy

$$\max(\deg(A_1), \deg(B_1)) \leq \tfrac{1}{2} \max(\deg(A), \deg(B)).$$

Also show that $A_1^4 - (\sqrt{i}B_1)^4 = A_1^4 + B_1^4$ is the square of a polynomial in $\mathbb{C}[t]$. Conclude that $A_1, \sqrt{i}B_1$ satisfy the conclusions of part c.

f. Conclude that if such a pair A, B exists, then one can repeat parts d and e infinitely many times with descreasing degrees at each step (this is the "infinite descent"). Explain why this is impossible and conclude that our original polynomials a, b, c, d must be constant.

Page 253, line 16: Replace "$x, w, \in$" with "$x, w \in$"

Page 253, line -10: Replace "subvarieties" with "subsets"

Page 254, line 14: Replace "is projection" with "be projection"

Page 254, line -13: Replace "I and V" with "I of V"

Page 255, line 12: Replace "$(c_2, \ldots, c_n)($" with "$(c_2, \ldots, c_n)$"

Page 255, line -14: Replace "nonvanishing at $(b_2, \ldots, b_n)$" with "nonvanishing at $(b_1, \ldots, b_n)$"

Page 255, line -6: Replace "$u - r^{N_1} f$" with "$u_r^{N_1} f$"

Page 255, line -4: Replace "$u_r^N f$" with "$u_r^{N_1} f$"

Page 256, line 2: Replace "$+ \cdots v_{j,r-1} x_1^{r-1}$" with "$+ \cdots + v_{j,r-1} x_1^{r-1}$"

Page 256, line -6: Replace "and that fact that" with "and the fact that"

Page 257, line 13: Replace "the $l = 1$ case of the Closure Theorem" with "the Extension Theorem"

Page 257, line 14: Replace "there is some $(c_1, \ldots, c_n) \in V$" with "$(c_1, \ldots, c_n) \in V$ for some $c_1 \in \mathbb{C}$"

Page 257, line -13: Replace "$k^{n-1} \to k^{n-1}$" with "$k^{n-1} \to k^{n-l}$"

Page 257, line -6: Replace "k^{n-1}" with "k^{n-l}"

Page 257, line -4: Replace "$\pi_l(V_1') \cup \cdots \cup \pi_l(V_m')$" with "$\pi_l(V_1) \cup \cdots \cup \pi_l(V_m)$"

Page 258, lines 3–16: Replace these lines with the following:

into irreducibles. This need not be a minimal decomposition, and in fact $V_i' = V_j'$ can occur when $i \neq j$. But we can find at least one of them not strictly contained in the others. By relabeling, we can assume $V_1' = \cdots = V_r'$ and $V_1' \not\subset V_i$ for $r + 1 \leq i \leq m$.

Applying (2) to the irreducible varieties $V_1, \ldots, V_r$ (with $W_0 = \emptyset$), there are varieties $W_i \subsetneqq V_i'$ such that

$$V_i' - W_i \subset \pi_l(V_i), \quad 1 \leq i \leq r$$

since V_i' is the Zariski closure of $\pi_l(V_i)$. If we let $W = W_1 \cup \cdots \cup W_r \cup V_{r+1}' \cup \cdots \cup V_m'$, then $W \subset \mathbf{V}(I_l)$, and one sees easily that

$$\begin{aligned} \mathbf{V}(I_l) - W &= V_1' \cup \cdots \cup V_m' - (W_1 \cup \cdots \cup W_r \cup V_{r+1}' \cup \cdots \cup V_m') \\ &\subset (V_1' - W_1) \cup \cdots \cup (V_r' - W_r) \\ &\subset \pi_l(V_1) \cup \cdots \cup \pi_l(V_r) \subset \pi_l(V). \end{aligned}$$

It remains to show that $W \neq \mathbf{V}(I_l)$. But if W were equal to $\mathbf{V}(I_l)$, then we would have $V_1' \subset W_1 \cup \cdots \cup W_r \cup V_{r+1}' \cup \cdots \cup V_m'$. Since V_1' is irreducible, Exercise 5 below shows that V_1' would lie in one of $W_1, \ldots, W_r, V_{r+1}', \ldots, V_m'$. This is impossible by the way

Page 258, line -14: Replace "First let" with "If $V = \emptyset$, then we are done. Otherwise let"

Page 259, line 14: Insert two spaces between the period "." and the word "Use"

Page 260, line 1: Replace "$\pi_l(V)$" with "$\pi_1(V)$"

Page 260, line -3: Replace "$\in z_1\}$" with "$\in Z_1\}$"

Page 275, line -11: In (2), replace "s_1^2+" with "s_1^2-"

Page 276, line -10: In (3), replace "$s_1^2 + \dfrac{a^2b + b^3}{(a^2+b^2)} s_1+$" with "$s_1^2 - bs_1+$"

Page 276, line -1: Replace the right hand side of the equation with
"$\dfrac{b}{2} \pm \dfrac{|a|\sqrt{4-(a^2+b^2)}}{2\sqrt{a^2+b^2}}$"

Page 283, line -7: Replace "g_s" with "g_t"

Page 284, line 12: Replace "$d_1 \cdot d_2 \cdots d_m$" with "$d_1 \cdot d_2 \cdots d_M$"

Page 285, lines 9, 11 and 12: Replace "g_s" with "g_t"

Page 289, line 3: Replace "ND" with "ND :"

Page 289, line 4: Replace "NC" with "NC :"

Page 291, line -6: Replace "Example 2" with "Example 3"

Page 293, line 3: Replace "**Example (continued)**" with "**Example 1 (continued)**"

Page 293, line 13: Replace "$\in$" with "$\subset$"

Page 293, line -11: In the formula for f_4, replace "$-u_4u_3$" with "$-u_1u_3$"

Page 293, line -8: Replace "$V = V(h_1, h_2, h_3, h_4) = V(f_1, \ldots, f_6)$" with "$V = \mathbf{V}(h_1, h_2, h_3, h_4) = \mathbf{V}(f_1, \ldots, f_6)$"

Page 298, line 10: Replace "$p \geq 1$" with "$p \geq 2$"

Page 299, line -7: Replace "$\mathbf{V}_{\mathbb{C}}$" with "$V_{\mathbb{C}}$"

Page 300, line 2: Replace "$g^s = \sum_{i=1}^n$" with "$g^s = \sum_{j=1}^n$"

Page 300, line 5: Replace "$\tilde{I} = \langle h_1, h_2, h_3, 1 - yg\rangle$" with "$\overline{I} = \langle h_1, h_2, h_3, h_4, 1 - yg\rangle$"

Page 300, line -2: Replace "$\mathbf{V}_{\mathbb{C}}$" with "$V_{\mathbb{C}}$"

Page 300, line -1: Replace "$V'_{\mathbb{C}}$" with "$V_{\mathbb{C}}$"

Page 301, line -5: Replace "*Steiner point*" with "*Steiner point* or *Fermat point*"

Page 305, line 11: Replace "that on polynomial" with "that one polynomial"

Page 305, line 20: Replace "LC" with "LC"

Page 308, line -3: Replace "$d_8 = -2x_4 \neq 0$" with "$d_8 = 2x_4 \neq 0$"

Page 309, line 2: Replace "condition $-2x_4 \neq 0$" with "condition $2x_4 \neq 0$"

Page 312, line 5: Replace "*we define*" with "*we define the* **elementary symmetric functions**"

Page 312, line -17: Replace "$\sigma_1 X^{n-2}$" with "$\sigma_1 X^{n-1}$"

Page 318, line 6: Replace "Theorems 3 and 6" with "Theorems 3 and 8"

Page 318, line -6: Replace "$f = x^n + a_1 x^{n-1} + \cdots + a_0$" with "$f = x^n + a_1 x^{n-1} + \cdots + a_n$"

Page 319, line 1: Replace "$f = x^n + a_1 x^{n-1} + \cdots + a_0$" with "$f = x^n + a_1 x^{n-1} + \cdots + a_n$"

Page 325, line 2: Replace "variants" with "invariants"

Page 327, line 2 of Exercise 3: Replace "$[I_n, A, A^2, \ldots, A^{m-1}]$" with "$\{I_n, A, A^2, \ldots, A^{m-1}\}$"

Page 327, line -14: Replace "rotations or" with "rotations of"

Page 329, line 23: Replace "$\in \mathrm{GL}(2, k)$" with "$\subset \mathrm{GL}(2, k)$"

Page 331, line -18: Replace "Proposition 11" with "Lemma 11"

Page 332, line -11: Replace "$a_1 A_1 \cdot \mathbf{x}$" with "$u_1 A_1 \cdot \mathbf{x}$"

Page 333, fifth line of the table in the middle of the page: In the last column, replace "$y^3 x$" with "xy^3"

Page 333, line -3: Replace "$k[x, y]$" with "$k[x, y]^{C_8}$"

Page 334, line -12: Replace "$+ \cdots A_t g_t + g$" with "$+ \cdots + A_t g_t + g$"

Page 334, line -11: Replace "$A_1, \ldots A_t, g$" with "$A_1, \ldots, A_t, g$"

Page 334, line -3: Replace "$C_n \cdot (f_m - y_m)$" with "$C_m \cdot (f_m - y_m)$"

Page 335, line 7: Replace "$C'_1 \cdot (f_m - y_m)$" with "$C'_m \cdot (f_m - y_m)$"

Page 336, line 15: Replace "integer: Hint: Use" with "integer. Hint: Use"

Page 337, line 21: Replace "$R_{C_8}(x^i, y^j)$" with "$R_{C_8}(x^i y^j)$"

Page 339, line 18: Replace "$|g|$" with "$[g]$"

Page 341, line 8: Replace "$x^3 y + xy^3$" with "$x^3 y - xy^3$"

Page 342, line 5 of the proof of Proposition 7: Replace "opposition inclusion" with "opposite inclusion"

Page 342, line -12: Replace "of Chapter 5, §5 to give" with "of Chapter 4, §5 to give"

Page 344, line -13: Replace "$\mathbf{V}(J_F \cap k[x_{i+1}, \ldots, x_n, y_1, \ldots, y_m]$" with "$\mathbf{V}(J_F \cap k[x_{i+1}, \ldots, x_n, y_1, \ldots, y_m])$"

page 344, line -18: Replace "$J_F \cap k[x_1, \ldots, x_n, y_1, \ldots, y_m]$" with "$J_F \cap k[x_i, \ldots, x_n, y_1, \ldots, y_m]$"

Page 345, line -18: Replace "$h_1(y_1, \ldots, y_m)x_i^{N-1} + \ldots + h_N(y_1, \ldots, y_m)$" with "$h_1(y_1, \ldots, y_m)x_i^{N-1} + \cdots + h_N(y_1, \ldots, y_m)$"

Page 347, part c of Exercise 8: Replace the Hint with the following: "Hint: If $p(f_1, f_2, f_3, f_4) = 0$ is a relation, use part a to reduce to a relation of the form $p_1(f_1, f_2, f_3) + p_2(f_1, f_2, f_3)f_4 = 0$. Then explain how degree arguments imply $p_1(f_1, f_2, f_3) = 0$."

Page 358, line 3: Replace "part b" with "part a"

Page 358, line 6: Replace "cases b and c" with "cases a and b"

Page 358, line 14: Replace "the asymptotes of" with "the points at ∞ corresponding to the asymptotes of"

Page 359, lines -15 and -16: Replace "$Ax - By - Cz = 0$" with "$Ax + By + Cz = 0$"

Page 362, line 18: Replace "$p \in \mathbb{P}^n$" with "$p \in \mathbb{P}^n(k)$"

Page 363, line 2: Replace "any homogeneous" with "any nonzero homogeneous"

Page 363, line 10: Replace "single homogeneous" by "single nonzero homogeneous"

Page 366, line 12: Replace "W and V" with "W in V"

Page 367, line -18: Replace "you reasoning" with "your reasoning"

Page 368, line 8: Replace "Let $\mathbb{P}^2(k)$ have" with "Let $\mathbb{P}^2(\mathbb{R})$ have"

Page 368, line 12: Replace "$\mathbb{P}^4$ and find" with "$\mathbb{P}^4(k)$ and find"

Page 368, line 17: Replace "$g_j(x_1, \ldots, x_{i-1}, x_{i-1}, \ldots, x_n) = f(x_1, \ldots, x_{i-1}, 1, x_i, \ldots, x_n)$," with "$g_j(x_1, \ldots, x_{i-1}, x_i, \ldots, x_n) = f(x_1, \ldots, x_i, 1, x_{i+1}, \ldots, x_n)$."

Page 368, line 18: Delete "where the 1 is substituted for x_i in f_j."

Page 369, line 5: Replace "p in $\mathbb{P}^n$" and "p in $\mathbb{P}^n(k)$"

Page 369, line -15: Replace "set on the right" with "set on the left"

Page 371, line -11: Replace "f_j" with "F_j"

Page 373, line 1: Replace "arbitrary field" with "infinite field"

Page 373, line -10: Replace "Now" with "If $f \neq 0$,"

Page 375, line 6: Replace "I_a" with "$\mathbf{I}_a$"

Page 376, line -1: Replace "$\sum_{i-j=k} f_i \cdot g_j$" with "$\sum_{i+j=k} f_i \cdot g_j$"

Page 377, lines -24 and -23: Replace "thecorresponding" with "the corresponding"

Page 377, line -22: Replace "letV_i" with "let V_i"

Page 377, line -3: Replace "There are some" with "There is some"

Page 378, line 7: Insert the new sentence "Let k be algebraically closed." at the beginning of part b of Exercise 11.

Page 381, line -13: In the first line of the display, replace "$\cup U_0$" with "$\cap U_0$"

Page 383, line 7: Replace "with respect to $k[x_1, \ldots, x_n]$" with "with respect to $>$

Page 383, line 16: Replace "is greater than" with "is smaller than"

Page 383, line -16: Replace "Exercise 10 of §3" with "Exercise 11 of §3"

Page 385, line -13: Replace "$(y^2-1)x-1$" with "$(y^2-1)x+1$"

Page 388, line 11: Replace "$v+vy$" with "vy^2+vy"

Page 388, line 12: Replace "$uy+uy^2$" with "$u+uy$"

Page 390, line 14: Replace "some I and β_i" with "some i and β_i"

Page 391, line 14: Replace "given Theorem 6" with "given in Theorem 6"

Page 391, line 21: In the display, replace "$0 \le I \le n$" with "$0 \le i \le n$"

Page 392, line 2: In the display, replace "$F^{(i)}(x_0,\ldots,1,\ldots,x_n,y_1,\ldots,y_m) \in$" with "$F^{(i)} = F(x_0,\ldots,1,\ldots,x_n,y_1,\ldots,y_m) \in$"

Page 394, line 17: Replace "$V(I_n)$" with "$\mathbf{V}(I_n)$"

Page 395, line 12: Replace "$V(g_1,\ldots,g_s)$" with "$\mathbf{V}(g_1,\ldots,g_s)$"

Page 396, line 8: Replace "compute a Groebner basis" with "compute a reduced Groebner basis"

Page 397, lines -23 and -22: Replace "of $(x_0,\ldots,x_n)$-homogeneous polynomials" with "$\sum_i F_i$, where F_i is a $(x_0,\ldots,x_n)$-homogeneous polynomial of degree i in $x_0,\ldots,x_n$"

Page 398, lines -4: Replace "polynomials" with "polynomials $\sum_i f_i$, where f_i is weighted homogeneous of weight i"

Page 399, line 10: Replace "$k[x_0,\ldots,x_n,y_1,\ldots,y_m]$" with "$k[x_0,\ldots,x_n,y_0,\ldots,y_m]$"

Page 403, line -7: Replace "form a" with "form an"

Page 404, line 15: Replace "part (a)" with "part (i)"

Page 405, line 7: Replace "Proposition 5" with "Proposition 6"

Page 405, line -14: Replace "by a Cartesian product $V \times \mathbb{P}^{n-p}$, where V is a nonsingular quadric in $\mathbb{P}^{p+1}$" with "as the join of a nonsingular quadric in $\mathbb{P}^p$ with a copy of $\mathbb{P}^{n-p-1}$"

Page 409, line 13: Replace "that $\mathbb{P}^3$" with "that in $\mathbb{P}^3$"

Page 410, line 2 of Exercise 6: Replace "$\mathbf{x}^t Q(\mathbf{x})$" with "$\mathbf{x}^t Q\mathbf{x}$"

Page 411, line 13: Replace "*singular point* if $\mathbf{V}(f)$" with "*singular point* of $\mathbf{V}(f)$"

Page 411, Exercise 10: Replace with the following new exercise:

10. Let $\mathbf{V}(f) \subset \mathbb{P}^n$ be a quadric of rank $p+1$, where $0 < p < n$. Prove that there are $X, Y \subset \mathbb{P}^n$ such that (1) $X \simeq \mathbf{V}(g) \subset \mathbb{P}^p$ for some nonsingular quadric g, (2) $Y \simeq \mathbb{P}^{n-p-1}$, (3) $X \cap Y = \emptyset$, and (4) $\mathbf{V}(f)$ is the *join* $X * Y$, which is defined to be the set of all lines in $\mathbb{P}^n$ connecting a point of X to a point of Y (and if $X = \emptyset$, we set $X * Y = Y$). Hint: Use Theorem 4.

Page 411, lines -21, -14 (twice) and -10: Replace "k^{n-1}" with "k^{n+1}" (total of four times)

Page 411, line -6: Replace "is a line" with "is either empty or a line"

Page 412, line 10: Replace "Theorem 10" with "Theorem 11"

Page 412, line -15: Replace "a line in $\mathbb{P}^5$" with "a line in $\mathbb{P}^3$"

Page 415, line 9: Replace "in the sense §3" with "in the sense of §3"

Page 415, line -3: Replace "C" with "$\mathbf{V}(f)$"

Page 416, line 2: Replace "$C - \mathbf{V}(f)$" with "$C = \mathbf{V}(f)$"

Page 421, line 17: Replace "$(4/7, -8/7)$" with "$(4/7, -8/7, 1)$"

Page 422, line -20: Replace "$\dfrac{\partial^{i+j} G}{\partial x^i y^j}$" with "$\dfrac{\partial^{i+j} G}{\partial x^i \partial y^j}$"

Page 422, line -13: Replace "$|z| = a^2 + b^2$" with "$|z| = \sqrt{a^2 + b^2}$"

Page 424, line 11: Replace "$L_1 \cup L_2 \cup L_3$ and $C_2 = L_4 \cup L_5 \cup L_6$" with "$L_1 \cup L_3 \cup L_5$ and $C_2 = L_2 \cup L_4 \cup L_6$"

Page 424, line -19: Replace "an equation of at most 2" with "an equation of degree at most 2"

Page 426, line 16: Replace "$F \to \mathbb{R}$" with "$F : U \to \mathbb{R}$"

Page 426, line 19: Replace "if $f \in \mathbb{C}[x_1, \ldots, x_n]$ is path connected" with "if $f \in \mathbb{C}[x_1, \ldots, x_n]$ is nonzero"

Page 430, line 14: Replace "a_j" with "α_j"

Page 430, line -14: Replace "or coordinate hyperplanes" with "of coordinate hyperplanes"

Page 430, line -6: Replace "provisional definition." with "provisional definition. We will always assume that k is infinite."

Page 431, line -15: Replace "$J = \{i_1, \ldots, i_n\}$" with "$J = \{i_1, \ldots, i_r\}$"

Page 432, line -10: Replace "where V_1 is a" with "where V_i is a"

Page 435, line 2: Replace "$ls - (1 + 2 + \cdots + l - 1)$" with "$l(s + 1) - (1 + 2 + \cdots + l - 1)$"

Page 435, line -13: Replace "$\{x^i y^j;$" with "$\{x^i y^j :$"

Page 435, line -11: Replace "$1 \le j \le l - 1$" with "$0 \le j \le l - 1$"

Page 436, line -3: Replace "$V(x_i :$" with "$\mathbf{V}(x_i :$"

Page 437, line 3: Replace "$x_{i_1}^{\alpha_1} \cdots x_{i_r}^{\alpha_r}$" with "$x_{i_1}^{\alpha_{i_1}} \cdots x_{i_r}^{\alpha_{i_r}}$"

Page 437, line 11: Replace "$V(x_i :$" with "$\mathbf{V}(x_i :$"

Page 440, line -20: Replace "d is s" with "d in s"

Page 443, line 7: Replace "$ls - (1 + 2 + \cdots + l - 1)$" with "$l(s + 1) - (1 + 2 + \cdots + l - 1)$"

Page 443, line -9: Replace "$[e_i, e_j] \in C(I)$" with "$[e_i, e_j] \subset C(I)$"

Page 443, line -6: Replace "finite number" with "finite nonzero number"

Page 444, line -9: Replace "all integer s" with "all integers s"

Page 445, first display: Replace "(2)" on left with "(3)"

Page 445, third display: Replace "(3)" on left with "(4)"

Page 445, line after third display: Replace "(2)" and "(3)" with "(3)" and "(4)" respectively

Page 445, part d of Exercise 11: Replace "(3)" and "(2)" with "(4)" and "(3)" respectively

Page 449, lines 9 and 11: Replace "$s \ge 5$" with "$s \ge 3$" (twice)

Page 449, line -14: Replace "$\mathbf{V}(I) = \mathbf{V}(\sqrt{I})$" with "$\sqrt{I}$ is monomial by Exercise 9 of §1 and $\mathbf{V}(I) = \mathbf{V}(\sqrt{I})$"

Page 449, line -9: Replace "$\langle \mathrm{LT}(\sqrt{I})\rangle$" with "$\mathrm{LT}(\sqrt{I})$"

Page 450, line 9: Insert the new sentence "We will always assume that the field k is infinite."

Page 450, line -13: Replace "part (iii) or Proposition 3" with "part (iii) of Proposition 3"

Page 451, line 21: Replace "maximal dimension of d of" with "maximal dimension d of"

Page 451, line 23: Replace "'Theorem 10" with "Theorem 8"

Page 451, line -9: Insert the new sentence "As above, we assume that k is infinite."

Page 453, line 9: Insert the new sentence "(Note that I is homogeneous by Proposition 4 of Chapter 8, §3.)"

Page 453, line -2: Replace "(see §4)" with "(see §4 of Chapter 8)"

Page 455, line -17: Replace "$k[x, y]$" with "$k[x, y, z]$"

Page 456, line 12: Replace "homogeneous polynomials" with "homogeneous ideals"

Page 456, line -22: Replace "$x_{i_1}, \ldots, x_{i_j}$" with "$x_{i_1}, \ldots, x_{i_t}$"

Page 456, line -18: Replace "$\deg {}^aHP_I$ has degree 1" with "$\deg {}^aHP_I = 1$"

Page 457, line -17: Replace "that ${}^aHF_I(s) =$" with "that $HF_I(s) =$"

Page 457, line -6: Replace "We first observe the following." with "As in §3, we assume that the field k is infinite."

Page 457, line -5: Delete "over an arbitrary field k"

Page 458, line 19: Replace "following proposition" with "following theorem"

Page 458, line -10: Replace "Theorem 13" with "Theorem 11"

Page 458, line -2: Replace "$k[x_1, \ldots, x_n]_s/I_s$" with "$k[x_0, \ldots, x_n]_s/I_s$"

Page 459, line 2: Replace "in $k[x_0, \ldots, x_n]_s/I_s$" with "in $k[x_0, \ldots, x_n]_s/(I + \langle f\rangle)_s$"

Page 459, line 13: In this display, the letters k, x, I, f should be in math italic, and the same is true for the letters n, s, f and r which appear in subscripts. Thus the display should read:

$$\dim k[x_0, \ldots, x_n]_s/I_s = \dim \alpha_f(k[x_0, \ldots, x_n]_{s-r}/I_{s-r}) + \dim k[x_0, \ldots, x_n]_s/(I + \langle f\rangle)_s$$

Page 459, line -14: Replace "Exercise 16" with "Exercise 15"

Page 459, lines -13 and -12: Replace "HF" with "HP" (twice)

Page 459, line -1: Replace the subscript "s" with "$s - r$" two times

Page 460, line 1: Replace the subscript "s" with "$s - r$" three times

Page 460, line -1: Replace "a_i" with "a_j"

Page 461, line -13: Replace "Corollary 4" with "Theorem 3"

Page 462, lines 2 and 3: Delete the sentences "If k is a finite field, V, W and $V \cup W$ are finite and, hence have dimension 0 by Proposition 6. Se we can assume that k is infinite."

Page 462, line 10: Replace "$\dim(V \cup W =$" with "$\dim(V \cup W) =$"

Page 462, line 16: Replace "$\langle \text{LT}(IJ)\rangle = \langle \text{LT}(I)\rangle \cdot \langle \text{LT}(J)\rangle$" with "$\langle \text{LT}(IJ)\rangle \supset \langle \text{LT}(I)\rangle \cdot \langle \text{LT}(J)\rangle$"

Page 462, line 18: Replace "$\mathbf{V}(\langle \text{LT}(IJ)\rangle) = \mathbf{V}(\langle \text{LT}(I)\rangle) \cup \mathbf{V}(\langle \text{LT}(J)\rangle$" with "$\mathbf{V}(\langle \text{LT}(IJ)\rangle) \subset \mathbf{V}(\langle \text{LT}(I)\rangle) \cup \mathbf{V}(\langle \text{LT}(J)\rangle$"

Page 462, lines 20–22: Replace with the following:

as a result, a coordinate subspace contained in $\mathbf{V}(\langle \text{LT}(IJ)\rangle)$ lies in either $\mathbf{V}(\langle \text{LT}(I)\rangle)$ or $\mathbf{V}(\langle \text{LT}(J)\rangle)$. This implies $\dim(V \cup W) \leq \max(\dim V, \dim W)$. The opposite inequality follows from Proposition 1, and the proposition is proved. $\square$

Page 463, line −9: Replace "$k[x_1, \ldots, x_n]_{s-r}$" with "$k[x_0, \ldots, x_n]_{s-r}$"

Page 464, line 2: Replace "on V" with "on I"

Page 464, line 3: Replace "r" with "r"

Page 464, part b of Exercise 9: Replace "Show that all projective and affine hypersurfaces are complete intersections." with "Show that hypersurfaces are complete intersections when k is algebraically closed."

Page 464, line −5: Replace "$\mathbf{I}(W_i) + \mathbf{I}(\{p_i\}) = 1$" with "$1 \in \mathbf{I}(W_i) + \mathbf{I}(\{p_i\})$"

Page 465, line 3: In the binomial coefficient, replace "$d - 1$" with "$d - i$"

Page 465, line −16: Replace "$\langle \text{LT}(I)\rangle \cdot \langle \text{LT}(J)\rangle = \langle \text{LT}(IJ)\rangle$" with "$\langle \text{LT}(I)\rangle \cdot \langle \text{LT}(J)\rangle \subset \langle \text{LT}(IJ)\rangle$"

Page 465, line −8: Replace "$0 \leq I \leq m - 1$" with "$0 \leq i \leq m - 1$"

Page 466, line 7: Replace "Chapter 5, §1" with "Chapter 5, §2"

Page 467, line −5: Replace "$k[y_r, \ldots, y_r]_{\leq s}$" with "$k[y_1, \ldots, y_r]_{\leq s}$"

Page 469, lines 18 and 19: Replace "this follows from the affine version of Proposition 11 of §4" with "this follows from Corollary 3 since a coordinate change gives isomorphic varieties"

Page 471, line −17: Replace "$= [0]$" with "$= \{0\}$"

Page 471, lines −12 and −7: Replace "algebraically closed" with "infinite" (twice)

Page 472, line −7: Replace "$\lambda a_0, \ldots, \lambda a_n)$" with "$(\lambda a_0, \ldots, \lambda a_n)$"

Page 475, line −13: Replace "total degree at" with "total degree"

Page 476, line 14: In the first sum, replace "$\dfrac{\partial f}{\partial x_1}(p)v_i$" with "$\dfrac{\partial f}{\partial x_i}(p)v_i$"

Page 477, line 3: Replace "Chapter 3, §2" with "Chapter 3, §4"

Page 479, lines −4 and −3: Replace "g_1" with "g" (twice)

Page 482, line 17: Replace "the zero polynomial" with "constant"

Page 482, line 25: Replace "if follows" with "it follows" and "part c" with "part b"

Page 482, line −18: Replace "$a^p = b^p$" with "$a^p + b^p$"

Page 483, lines 18 and 19: Replace the sentence "If $V \subset k^n$ is a hypersurface ... a singular point at p." with "Let $V \subset k^n$ be a hypersurface with $\mathbf{I}(V) = \langle f \rangle$. Show that if V is not a hyperplane and $p \in V$ is nonsingular, then either the variety $V \cap T_p(V)$ has a singular point at p or the restriction of f to $T_p(V)$ has an irreducible factor of multiplicity ≥ 2."

Page 483, line 20: Replace "so that $T_p(V)$ is defined" with "so that $p = 0$ and $T_p(V)$ is defined"

Page 483, line 21: Add the new sentences "Then the restriction of f to $T_p(V)$ is the polynomial $f(0, x_2, \ldots, x_n)$. See also Example 4."

Page 483, line -19: Replace "f_r" with "f_{n-d}"

Page 483, line -4: Replace "tangent plane" with "tangent space"

Page 488, line 4: Replace "$v \in k^n$" with "$v \in \mathbb{C}^n$"

Page 488, line 6: Replace "$L \in \mathbb{C}^n$" with "$L \subset \mathbf{C}^n$"

Page 488, line -18: Replace "p in $\mathbb{C}^n$" with "$p \in V$"

Page 489, line 1: Replace "$t_k^i f(v_k)$" with "$t_k^i f_i(v_k)$"

Page 489, line 11: Replace "$vt \in V$" with "$tv \in V$"

Page 490, line -19: Replace "vt" with "tv"

Page 490, line -15: Replace "L is the secant line" with "L_k is the secant line"

Page 491, line -9: Replace "$\mathbb{C}^n$" with "k^n"

Page 492, line 4 of Exercise 1: Replace "$\dfrac{\partial^\alpha}{\partial^\alpha x} = \dfrac{\partial^{\alpha_1}}{\partial^{\alpha_1} x_1} \cdots \dfrac{\partial^{\alpha_n}}{\partial^{\alpha_n} x_n}$" with "$\dfrac{\partial^\alpha}{\partial x^\alpha} = \dfrac{\partial^{\alpha_1}}{\partial x_1^{\alpha_1}} \cdots \dfrac{\partial^{\alpha_n}}{\partial x_n^{\alpha_n}}$"

Page 492, line 8 of Exercise 1: Replace "$\dfrac{\partial^\alpha (x-p)^\beta}{\partial^\alpha x}(p)$" with "$\dfrac{\partial^\alpha (x-p)^\beta}{\partial x^\alpha}(p)$"

Page 492, lines 12 and 14 of Exercise 1: Replace "$\dfrac{\partial^\alpha f}{\partial^\alpha x}(p)$" with "$\dfrac{\partial^\alpha f}{\partial x^\alpha}(p)$" in two places

Page 494, line 8: Replace "$\lambda \in \mathbb{C}$" with "$\lambda \in k$"

Page 495, line 6: Replace "$(\mathbb{P}^{n-1} \times (V - \{0\})$" with "$(\mathbb{P}^{n-1} \times (V - \{0\}))$"

Page 495, line 14: Replace "of V at p" with "of V at 0"

Page 495, lines 19 and 21: Replace "$g(tq, q) = 0$" with "$g(q, tq) = 0$" (twice)

Page 495, line 27: Replace "$g(tx_1, \ldots, tx_n, x_1, \ldots, x_n)$" with "$g(x_1, \ldots, x_n, tx_1, \ldots, tx_n)$"

Page 498, line -9: Replace "are $1 \in G$" with "is $1 \in G$"

Page 498, line -4: Replace "with entries k" with "with entries in k"

Page 499, line -7: Replace "Proposition 4" with "Proposition 8"

Page 500, line -2: Replace "Proposition 5" with "Proposition 9"

Page 505, line -3: Immediately after "years.", insert the new sentence "AXIOM is now freely available from `http://www.nongnu.org/axiom`."

Page 510, line 8: Insert a semicolon at the end of the line

Page 510, lines 12 and 13: Replace these lines with the following:

Finally, there is a Maple package written by Albert Lin and Philippe Loustaunau of George Mason University (with subsequent modifications by David Cox and Will Gryc of Amherst College and Chris Wensley of the University of Bangor, Wales) which extends the Groebner

Page 510, lines 19–21: Replace these lines with the following:

obtained electronically by sending email to `dac@cs.amherst.edu` or by going to the web site `http://www.cs.amherst.edu/~dac/iva.html`.

Page 510, line 21: After this line, insert the following new paragraph:

Starting with Release 5 of Maple V, the `grobner` package described above has been replaced with the new and improved `Groebner` package. A version of §2 of this Appendix for the `Groebner` package can be found at the web site `http://www.cs.amherst.edu/~dac/iva.html`.

Page 511, line 17: Replace "`2 * x * y`" with "`2 x y`"

Page 511, line −14: Replace "`{v * x^2 + y, u * x * y + y^2}`" with "`{v x^2 + y, u x y + y^2}`"

Page 512, lines 1 and 2: Replace these lines with the following:

Finally, there is a Mathematica package written by Susan Goldstine of Amherst College (with an update by Will Gryc, also of Amherst) which includes many commands relevant to the

Page 512, lines 8–10: Replace these lines with the following:

obtained electronically by sending email to `dac@cs.amherst.edu` or by going to the web site `http://www.cs.amherst.edu/~dac/iva.html`.

Page 515, line 16: Replace "Chapter 2, §2" with "Chapter 2, §4"

Page 533, index entry for monomial ordering: In the subentry for graded reverse lexicographic, replace "(grlex)" with "(grevlex)"

Page 533, index entry for polynomial: After the subentry for affine Hilbert, add a new subentry: "bihomogeneous, 396"

Page 534, index entry for PoSSo: replace "systenms" with "systems"